A Low Complexity Semi-Distributed Algorithm for MIMO Inteference Channel

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***Abstract*—This paper deals with the design of pre-coding and decorrelation matrices in the constant MIMO interference channel. First, we propose a distributed algorithm that enables all the streams of the links in the network to reach a set of target signal to interference plus noise ratios (SINRs). Depending on the chosen SINR targets, it is shown that the algorithm performs either perfect interference alignment at the receivers or tries to strengthen the useful signal regardless of interference. In a second step, we address the choice of the target SINRs and the complexity issue of the proposed distributed algorithm. We design a semi-distributed coordination algorithm that relies on a power optimization process to choose the best combination of target SINRs and to ensure a fast convergence. The performance of the proposed coordination algorithm in terms of system sum-capacity and complexity are compared to those of minimum leakage interference alignment and MAX-SINR algorithms already existing in the literature.**

I. Introduction

Today, the Long Term Evolution (LTE) system started to be deployed in several countries and the wireless networks are expected to see their capacity increasing. However, the emergence of new multimedia services and the wider use of smart phones make the traffic data rate increase continuously. To answer this demand, small cells that have lower power nodes are a serious candidate to be used in the near future to offload a part of the traffic. This needs also different level of cooperation between these nodes to coordinate their interferences in order to increase the system capacity. When only the exchange on the channel state information (CSI) is considered, the network having K pairs of transmitter-receiver could be modeled as a MIMO interference channel (MIMO-IC) [1], [2].

Recently [3] proposed a technique based on the interference alignment (IA) at each receiver served by one transmission node. It aims at confining all the interference from the interfering nodes into only one dimension. Authors showed that for high SINR regions the IA is capacity achieving. The alignment could be done not only in time or in frequency domains but also in space when the transmitter and receiver are using more than one antenna [4]–[6]. Several algorithms for alignments have been proposed depending on the local or partial or total CSI knowledge at the transmitter or receivers [7]–[16].

$^{\diamond}$ Part of this work has been performed in the framework of the FP7 project ICT-317669 METIS, which is partly funded by the European Union. The authors would like to acknowledge the contributions of their colleagues in METIS, although the views expressed are those of the authors and do not necessarily represent the project.

When the channel is assumed to be locally known at each node, iterative techniques for the interference alignments were proposed [11]–[16]. They are interesting in particular when the considered network doesn't have an architecture making a high, speed and reliable information exchange between its nodes. The minimum leakage and the Max SINR algorithms are shown to have high performance. Whereas, the former has a low convergence speed and the later convergence is not guaranteed.

In practical scenarios, the system SINRs are medium or low and this imply that the IA could not be the optimum strategies to use. In [15], an algorithm based on useful signal-IA tradeoff was proposed however, there was no practical rule to optimize the balancing parameters. In [17] we have proposed an algorithm introducing low level of cooperation between transmitting nodes to reach target SINRs at all receivers. It was based on optimum balancing rules and its convergence was proved. These algorithms assumed an equal power transmission at their transmission antennas.

In a first step of this work we propose a distributed algorithm enabling to reach a set of target signal to interference plus noise ratios (SINRs) at the different streams of the different links in the network. In a second step, we propose to allow a limited information exchange at a central unit (CU) level and to use the alleviate the complexity of the distributed algorithm and to adapt the choice of target SINRs. Only the information needed to check the existence of an optimum power allocation to reach a target SINRs are sent to the CU [18]. The CU computes this optimal power allocation when it exists. The proposed algorithm is shown to decrease the iteration number to reach the target SINRs and to have quasi-optimal performance.

This paper is organized as follows: : section II gives a general background and explains the interference alignment process. Section III deals with the distributed algorithm which aim at ensuring a set of target SINRs at the different streams of the different links in the network. Section IV is devoted to the semi-distributed coordination algorithm.

In the sequel, the following notations are used: capital and small bold letters stand for matrices and vectors. The superscripts $(.)^T$, $(.)^H$, and $(.)^{-1}$ denote the transpose, the conjugate transpose and the inverse of a given matrix, respectively. $\mathbf{0}$ and $\mathbf{I}$ denote the all zero and identity matrices, respectively. $\nu^R_{min}(\mathbf{A})$ and $\nu^R_{max}(\mathbf{A})$ are matrices whose columns are the eigenvectors corresponding to the R smallest (the greatest, respectively) eigenvalues of matrix $\mathbf{A}$. We use $|\mathbf{A}|$ to denote the determinant of a square matrix $\mathbf{A}$ and $||\mathbf{A}||_F$ to denote the Frobenius norm of matrix $\mathbf{A}$. We use $\mathbf{A}^{[\star l]}$ and $\mathbf{A}^{[l\star]}$ to denote the l^{th} column and the l^{th} row of matrix $\mathbf{A}$, respectively. $\mathbf{A}^{[ij]}$ denotes the entry (i,j) of matrix $\mathbf{A}$. $\text{diag}(a_1,\ldots,a_n)$ is a diagonal matrix with a_i is the diagonal item number i. Finally, $\text{E}[.]$ denotes expectation

II. General Background

A. System Model

We consider a system of K APs equipped with M_T transmit antennas and K users equipped with M_R receive antennas. Each AP is paired with a single user in a one to one mapping. Without loss of generality, we assume that both of them has index k. Each AP interferes with all the receivers it is not paired with. The system represents, as such, the K-user MIMO interference channel. The channels between the different links are considered to be narrow-band where each link is static for the duration of a transmission but may change between successive transmissions. This is the block-fading model, where all the links in the network are constant for the period of transmission, creating a tractable approximation to more realistic continuous-fading models. Finally, local and perfect channel knowledge is assumed at each of the transmit and receive nodes.
The transmitted signal $\mathbf{x}_k \in \mathbb{C}^{M_T\times 1}$ from AP k to its attached user is given by

$$\mathbf{x}_k = \mathbf{V}_k\mathbf{s}_k, \tag{1}$$

where $\mathbf{V}_k \in \mathbb{C}^{M_T\times d_k}$ is a pre-coding matrix applied to the streams vector intended to user k, d_k is the number of streams transmitted on link k and $\mathbf{s}_k \in \mathbb{C}^{d_k\times 1}$ is the vector containing the data symbols intended to user k and verifying $\mathbf{Q}_{kk} = \text{E}\big[\mathbf{s}_k\mathbf{s}_k^H\big] = \text{diag}(P_1^k,\ldots,P_{d_k}^k)$ with P_i^k is the power allocated to stream i of link k and respecting the power constraint $\sum_{i=1}^{d_k} P_i^k \leq P$ with P is the total power available at every AP. Recall that in case of equal power allocation, $P_i^k = \frac{P}{d_k} \quad \forall i = 1,\ldots,d_k,\ \forall k$.
The received signal $\mathbf{y}_k \in \mathbb{C}^{M_R\times 1}$ by user k is given by

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{V}_k\mathbf{s}_k + \sum_{k'\neq k}\mathbf{H}_{kk'}\mathbf{V}_{k'}\mathbf{s}_{k'} + \mathbf{n}_k, \tag{2}$$

where $\mathbf{H}_{kk'} \in \mathbb{C}^{M_R\times M_T}$ is the channel matrix between user k and AP k' and $\mathbf{n}_k \in \mathbb{C}^{M_R\times 1}$ is an additive complex white Gaussian noise vector verifying $\text{E}\big[\mathbf{n}_k\mathbf{n}_k^H\big] = \sigma^2\mathbf{I}$.
At the reception, each user k decorrelates the received signal by applying the decorrelation matrix $\mathbf{U}_k \in \mathbb{C}^{d_k\times M_R}$, which leads to the following relation

$$\mathbf{U}_k\mathbf{y}_k = \mathbf{U}_k\mathbf{H}_{kk}\mathbf{V}_k\mathbf{s}_k + \mathbf{U}_k\sum_{k'\neq k}\mathbf{H}_{kk'}\mathbf{V}_{k'}\mathbf{s}_{k'} + \mathbf{U}_k\mathbf{n}_k. \tag{3}$$

The dual system is described using left arrow notations. The dual channel is hence defined as

$$\overleftarrow{\mathbf{y}}_k = \overleftarrow{\mathbf{H}}_{kk}\overleftarrow{\mathbf{V}}_k\overleftarrow{\mathbf{s}}_k + \sum_{k'\neq k}\overleftarrow{\mathbf{H}}_{kk'}\overleftarrow{\mathbf{V}}_{k'}\overleftarrow{\mathbf{s}}_{k'} + \overleftarrow{\mathbf{n}}_k, \tag{4}$$

where $\overleftarrow{\mathbf{y}}_k \in \mathbb{C}^{M_T\times 1}$ is the received signal by receiver k, $\overleftarrow{\mathbf{H}}_{kk'} = \mathbf{H}_{k'k}^H$ is the channel between transmitter k' and receiver k, $\overleftarrow{\mathbf{V}}_k \in \mathbb{C}^{M_R\times d_k}$ is a pre-coding matrix applied to the streams vector intended to user k, $\overleftarrow{\mathbf{s}}_k \in \mathbb{C}^{d_k\times 1}$ is the vector containing the data symbols intended to user k, and $\overleftarrow{\mathbf{n}}_k \in \mathbb{C}^{M_T\times 1}$ is an additive complex white Gaussian noise vector verifying $\text{E}\big[\overleftarrow{\mathbf{n}}_k\overleftarrow{\mathbf{n}}_k^H\big] = \sigma^2\mathbf{I}$. Finally, the decorrelation matrix at receiver k is denoted by $\overleftarrow{\mathbf{U}}_k$.

B. IA over Constant MIMO Interference Channel

The IA technique for constant MIMO IC aims at designing the pre-coding and decorrelation matrices in eq (1) and eq (3) so that they satisfy the following two conditions:

$$\mathbf{U}_k\mathbf{H}_{kk'}\mathbf{V}_{k'} = \mathbf{0}\ \forall k \neq k', \tag{5}$$
$$\text{rank}\big(\mathbf{U}_k\mathbf{H}_{kk}\mathbf{V}_k\big) = d_k. \tag{6}$$

Eq (7) reflects the fact that at every receiver we try to confine the interfering signals into a reduced dimension subspace. Eq (8) reflects the fact that we leave some dimension to acquire the useful signal.

III. Proposed Distributed Algorithm

A. Algorithm Principle

The proposed distributed algorithm exploits a duality result from [19] which states that for the MIMO IC, if a set target SINRs is achievable in the downlink (DL) direction (under a sum-power constraint), these target SINRs will be achievable in the reverse direction under the same power constraint. The aim of the design is to ensure a set of SINRs $\gamma_1^1,\gamma_1^2,\ldots,\gamma_1^{d_1},\ldots,\gamma_K^1,\gamma_K^2,\ldots,\gamma_K^{d_K}$ at all the streams of all the links.

B. Algorithm Description

Assuming equal power allocation between the different streams, let us define the SINR Γ_k^l of stream l of user k by

$$\Gamma_k^l = \frac{\mathbf{U}_k^{[l\star]}\mathbf{H}_{kk}\mathbf{V}_k^{[\star l]}\mathbf{V}_k^{[\star l]H}\mathbf{H}_{kk}^H\mathbf{U}_k^{[l\star]H}\frac{P}{d_k}}{\sigma^2 + I_{kl}^{InterStream} + I_{kl}^{InterLink}}, \tag{7}$$

with

$$I_{kl}^{InterStream} = \frac{P}{d_k}\mathbf{U}_k^{[l\star]}\Big(\sum_{l'\neq l}\mathbf{H}_{kk}\mathbf{V}_k^{[\star l']}\mathbf{V}_k^{[\star l']H}\mathbf{H}_{kk}^H\Big)\mathbf{U}_k^{[l\star]H}, \tag{8}$$

$$I_{kl}^{InterLink} = \frac{P}{d_{k'}}\mathbf{U}_k^{[l\star]}\Big(\sum_{k\neq k}\sum_{l'}\mathbf{H}_{kk'}\mathbf{V}_{k'}^{[\star l']}\mathbf{V}_{k'}^{[\star l']H}\mathbf{H}_{kk'}^H\Big)\mathbf{U}_k^{[l\star]H}. \tag{9}$$

The goal of the design will be to achieve the following conditions for all the streams of all the links:

$$\Gamma_k^l \geq \gamma_k^l \quad \forall k, \forall l. \tag{10}$$

Therefore, in order to allow each stream of each user reach its target SINR γ_k^l, eq (7) and eq (10) yield

$$\Big|\Big|(\frac{1}{\gamma_k^l}+1)\frac{1}{d_k}\mathbf{U}_k^{[l\star]}\mathbf{H}_{kk}\mathbf{V}_k^{[\star l]}\Big|\Big|_F^2 - \sum_{k'}\sum_{l'}\Big|\Big|\frac{1}{d_{k'}}\mathbf{U}_k^{[l\star]}\mathbf{H}_{kk'}\mathbf{V}_{k'}^{[\star l']}\Big|\Big|_F^2 \geq \frac{\sigma^2}{P} \tag{11}$$

Based on eq (11), and for a given target SINRs $\gamma_1^1, \gamma_1^2, \ldots, \gamma_1^{d_1}, \ldots, \gamma_K^1, \gamma_K^2, \ldots, \gamma_1^{d_K}$, at every iteration the decorrelation matrices are computed so that we have

$$\begin{aligned}\mathbf{U}_k^{[l\star]} &= \arg\max_{\tilde{\mathbf{U}}_k} \left\|\left(\frac{1}{\gamma_k^l}+1\right)\frac{1}{d_k}\tilde{\mathbf{U}}_k^{[l\star]}\mathbf{H}_{kk}\mathbf{V}_k^{[\star l]}\right\|_F^2 \\ &- \sum_{k'}\sum_{l'}\left\|\frac{1}{d_{k'}}\tilde{\mathbf{U}}_k^{[l\star]}\mathbf{H}_{kk'}\mathbf{V}_{k'}^{[\star l']}\right\|_F^2. \quad (12)\end{aligned}$$

Given fixed values of $\mathbf{U}_k$, at each iteration, the matrices $\mathbf{V}_k$ must verify

$$\begin{aligned}\mathbf{V}_k^{[l\star]} &= \arg\max_{\tilde{\mathbf{V}}_k} \left\|\left(\frac{1}{\gamma_k^l}+1\right)\frac{1}{d_k}\tilde{\mathbf{V}}_k^{[l\star]}\mathbf{H}_{kk}^H\mathbf{U}_k^{[l\star]H}\right\|_F^2 \\ &- \sum_{k'}\sum_{l'}\left\|\frac{1}{d_{k'}}\tilde{\mathbf{V}}_k^{[l\star]}\mathbf{H}_{k'k}^H\mathbf{U}_{k'}^{[l'\star]H}\right\|_F^2. \quad (13)\end{aligned}$$

For high values of $\gamma_1^1, \gamma_1^2, \ldots \gamma_1^{d_1}, \ldots, \gamma_K^1, \gamma_K^2, \ldots, \gamma_K^{d_K}$, the first terms in the maximized objective function in eq (12) and eq (13) become negligible compared to the second terms. Therefore, the proposed algorithm will simply become the minimum leakage distributed IA algorithm.

The proposed distributed algorithm operates as follows

Algorithm 1 Distributed joint signal and interference alignment

1: Start with arbitrarily pre-coding vectors $\mathbf{V}_1, \ldots, \mathbf{V}_K$ with linearly independent unit norm columns $\forall k = 1, 2, \ldots, K$

2: For each stream l of receiver k, compute the following matrix

$$\begin{aligned}\mathbf{Q}_{kl} &= \left(\frac{1}{\gamma_k^l}+1\right)\frac{1}{d_k}\mathbf{H}_{kk}\mathbf{V}_k^{[\star l]}\mathbf{V}_k^{[\star l]H}\mathbf{H}_{kk}^H \\ &- \sum_{k'}\sum_{l'}\frac{1}{d_{k'}}\mathbf{H}_{kk'}\mathbf{V}_{k'}^{[\star l']}\mathbf{V}_{k'}^{[\star l']H}\mathbf{H}_{kk'}^H\end{aligned}$$

3: For each stream l of receiver k, compute the decorrelation vector $\mathbf{U}_k^{[l\star]}$ which corresponds to the l^{th} row of matrix $\mathbf{U}_k$ and is given by

$$\mathbf{U}_k^{[l\star]H} = \nu_{max}^1(\mathbf{Q}_{kl})$$

4: Reverse the communication direction and set $\overleftarrow{\mathbf{V}}_k = \mathbf{U}_k^H$

5: For each stream l of new receiver k, compute the following matrix

$$\begin{aligned}\overleftarrow{\mathbf{Q}}_{kl} &= \left(\frac{1}{\gamma_k^l}+1\right)\frac{1}{d_k}\overleftarrow{\mathbf{H}}_{kk}\overleftarrow{\mathbf{V}}_k^{[\star l]}\overleftarrow{\mathbf{V}}_k^{[\star l]H}\overleftarrow{\mathbf{H}}_{kk}^H \\ &- \sum_{k'}\sum_{l'}\frac{1}{d_{k'}}\overleftarrow{\mathbf{H}}_{kk'}\overleftarrow{\mathbf{V}}_{k'}^{[\star l']}\overleftarrow{\mathbf{V}}_{k'}^{[\star l']H}\overleftarrow{\mathbf{H}}_{kk'}^H\end{aligned}$$

6: For each stream l of new receiver k, compute the decorrelation vector $\overleftarrow{\mathbf{U}}_k^{[l\star]}$ which corresponds to the l^{th} row of matrix $\overleftarrow{\mathbf{U}}_k$ and is given by

$$\overleftarrow{\mathbf{U}}_k^{[l\star]H} = \nu_{max}^1(\overleftarrow{\mathbf{Q}}_{kl})$$

7: Reverse the communication direction and set $\mathbf{V}_k = \overleftarrow{\mathbf{U}}_k^H$

8: Repeat the process until the convergence of the algorithm

The proposed distributed algorithm converges even if the target SINRs are not reachable by all the streams of all the links (the proof of the convergence is presented in the appendix).

IV. Semi-distributed Coordination Algorithm

The coordination is based on a power optimization process: the APs run the distributed algorithm presented in section III for a given set of target SINRs. After every iteration $Iter$, the APs feed back some control information to a CU. Based on these information, the CU checks whether there exists an optimal power allocation that enables to reach the chosen target SINRs and therefore there is no need to continue to iterate. The power allocation determination is based on an existing algorithm in [18]. The algorithm enables as such to save the number of iterations and consequently to reduce the implementation complexity.

Here the computations made at the CU level are explained. We are placed in the case of $d_k = 1\ \forall k$. However, the analysis can be generalized to the case of multiple streams per link. Based on the received control information from the APs, the CU constructs the matrices $\mathbf{\Psi}(Iter)$, $\mathbf{D}(Iter)$ with

$$\mathbf{\Psi}^{[ik]}(Iter) = \begin{cases}(\mathbf{V}_k^{Iter})^H\mathbf{R}_{ik}^{Iter}\mathbf{V}_k^{Iter} & i \neq k, \\ 0 & i = k.\end{cases} \quad (14)$$

$$\mathbf{D}(Iter) = \text{diag}\left\{\frac{\gamma_1}{(\mathbf{V}_1^{Iter})^H\mathbf{R}_{11}^{Iter}\mathbf{V}_1^{Iter}}, \ldots, \frac{\gamma_K}{(\mathbf{V}_K^{Iter})^H\mathbf{R}_{KK}^{Iter}\mathbf{V}_K}\right\}, \quad (15)$$

where

$$\mathbf{R}_{ik}^{Iter} = \mathbf{H}_{ik}^H(\mathbf{U}_i^{Iter})^H\mathbf{U}_i^{Iter}\mathbf{H}_{ik}, \quad (16)$$

and $\mathbf{V}_k^{Iter}$ and $\mathbf{U}_k^{Iter}$ are the pre-coding and decorrelation matrices obtained at iteration number $Iter$ of the distributed algorithm of section III.

Based on [18], for a given pre-coding and decorrelation matrices $\mathbf{V}_k^{Iter}$ and $\mathbf{U}_k^{Iter}$, if there exists an optimal power allocation enabling to reach target SINRs, at least one AP k_0 will transmit at full power P. Let k_0 be known a priori (k_0 can be found by hypothesis testing) and let us define the matrix $\mathbf{\Phi}(Iter)$ by

$$\mathbf{\Phi}(Iter) = \begin{pmatrix}\mathbf{D}(Iter)\mathbf{\Psi}(Iter) & \mathbf{D}(Iter)\sigma \\ \mathbf{1}_{k_0}\mathbf{D}(Iter)\mathbf{\Psi}(Iter)/P & \mathbf{1}_{k_0}\mathbf{D}(Iter)\sigma/P\end{pmatrix},$$

where $\mathbf{1}_{k_0}$ is a $K \times 1$ vector with one in position k_0 and zeros in other positions and σ is a $K \times 1$ vector with σ^2 in all positions.

The set of target SINRs is achievable iff the maximum eigenvalue $C(Iter, P)$ of matrix $\mathbf{\Phi}(Iter)$ is less or equal to 1. The optimal power allocation corresponds to the K first inputs of the eigenvector corresponding to the maximal eigenvalue of matrix $\mathbf{\Phi}(Iter)$. Let us denote $Iter_{convergence}$ the necessary number of iterations necessary for the convergence of the distributed algorithm of section III. The operation of the semi-distributed algorithm is detailed in what follows

Algorithm 2 Description of the semi-distributed algorithm

Choose a set of target SINRs $\gamma_1, \dots, \gamma_K$
$Iter = 1$
The APs compute $\mathbf{U}_k^{Iter}$ and $\mathbf{V}_k^{Iter}$ $\forall k$
The APs feed back the necessary control information to the CU
The CU computes $C(Iter, P)$
while $\big(Iter \neq Iter_{convergence}\big)$ or $\big(C(Iter, P) \gtrsim 1\big)$ **do**
    $Iter = Iter + 1$
    The APs compute $\mathbf{U}_k^{Iter}$ and $\mathbf{V}_k^{Iter}$ $\forall k$
    The APs feed back the necessary control information the CU
    The CU computes $C(Iter, P)$
end while
The CU informs each AP of the power that must be allocated to its served user
The APs transmit the data using the proper transmit/receive and power allocation parameters

A. Simulation Assumptions and Results

In this section, all the simulations are run considering the same assumptions on the channel coefficients of the previous section. The simulations are also considered for the case of the 3-user 2 × 2 MIMO IC.

For the semi-distributed coordination algorithm, the chosen target SINRs are the average SNR of every link. However other target SINR combinations can be adopted.

Fig.1 illustrates the average sum-capacity evolution of the proposed algorithms compared to the minimum leakage and MAX-SINR algorithms. The figure shows that the proposed algorithms outperform the minimum leakage algorithm in low and medium SNR regions and approaches the performance of the MAX-SINR algorithm. The same figure show that at high system SNR, the minimum leakage and MAX-SINR algorithms outperform the semi-distributed algorithm in terms of sum-capacity. Such behavior can be overcome if we use higher target SINRs as design parameter which will push it to imitate the minimum leakage algorithm behavior in these system SNR regions.

Fig.2 illustrates the number of required iterations for convergence for the proposed semi-distributed coordination algorithm and the minimum leakage algorithm. The evolution is drawn function of the system SNR. The figure shows that the proposed algorithms requires less iterations to converge compared to the minimum leakage algorithm. From another side, from the figure we notice that the proposed algorithm is the most advantageous in terms of implementation complexity seeing that it requires the least number of iterations to converge. Recall that no comparison is made with the MAX-SINR algorithm in terms of convergence speed seeing that its convergence is not always guaranteed.

V. Conclusion

In this paper, we proposed in a first step a distributed algorithm for the design of linear pre-coding and decorrelation matrix for the MIMO IC. In a second step we proposed a semi-distributed algorithm that optimise the power allocation to adapt the performance of the previously proposed distributed algorithm to the system SNR. Based on Monte-Carlo simulation, we showed that the proposed coordination algorithm outperform the minimum leakage algorithm proposed in [11] and have a close behavior to the MAX-SINR algorithm in terms of system capacity. From another side, the proposed algorithm is advantageous in terms of computational complexity compared to the minimum leakage algorithm.

Appendix
Proof of Theorem 1

The proof of the convergence of the proposed algorithm is presented for the case of K-user MIMO IC and for $d = 1\ \forall k$, the generalization to the case of $d \gtrsim 1$ is straightforward. Let us introduce the quantity T defined by

$$T = \sum_{k=1}^{K} \mathbf{U}_k \Big(\frac{1}{\gamma_T} \frac{P}{d} \mathbf{H}_{kk} \mathbf{V}_k \mathbf{V}_k^H \mathbf{H}_{kk}^H - \sum_{k' \neq k} \frac{P}{d} \mathbf{H}_{kk'} \mathbf{V}_{k'} \mathbf{V}_{k'}^H \mathbf{H}_{kk'}^H \Big) \mathbf{U}_k^H . \quad (17)$$

Given fixed value of $\mathbf{V}_1, \dots, \mathbf{V}_K$, we have

$$\max_{\mathbf{U}_1, \dots, \mathbf{U}_K} T = \sum_{k=1}^{K} \max_{\mathbf{U}_k} T_k, \quad (18)$$

with

$$T_k = \mathbf{U}_k \Big(\frac{1}{\gamma_T} \frac{P}{d} \mathbf{H}_{kk} \mathbf{V}_k \mathbf{V}_k^H \mathbf{H}_{kk}^H - \sum_{k' \neq k} \frac{P}{d} \mathbf{H}_{kk'} \mathbf{V}_{k'} \mathbf{V}_{k'}^H \mathbf{H}_{kk'}^H \Big) \mathbf{U}_k^H . \quad (19)$$

As such, given fixed values of $\mathbf{V}_1, \dots, \mathbf{V}_K$, the computed $\mathbf{U}_1, \dots, \mathbf{U}_K$ maximize T at every iteration. From another side, we have

$$\begin{aligned} T &= \sum_{k=1}^{K} \overleftarrow{\mathbf{V}}_k^H \Big(\frac{1}{\gamma_T} \frac{P}{d} \mathbf{H}_{kk} \overleftarrow{\mathbf{U}}_k^H \overleftarrow{\mathbf{U}}_k \mathbf{H}_{kk}^H - \sum_{k' \neq k} \frac{P}{d} \mathbf{H}_{kk'} \overleftarrow{\mathbf{U}}_{k'}^H \overleftarrow{\mathbf{U}}_{k'} \mathbf{H}_{kk'}^H \Big) \overleftarrow{\mathbf{V}}_k \\ &= \sum_{k=1}^{K} \overleftarrow{\mathbf{V}}_k^H \Big(\frac{P}{d\gamma_T} \mathbf{H}_{kk} \overleftarrow{\mathbf{U}}_k^H \overleftarrow{\mathbf{U}}_k \mathbf{H}_{kk}^H \Big) \overleftarrow{\mathbf{V}}_k - \sum_{k=1}^{K} \sum_{k' \neq k} \frac{P}{d} \overleftarrow{\mathbf{V}}_k^H \big(\mathbf{H}_{kk'} \overleftarrow{\mathbf{U}}_{k'}^H \overleftarrow{\mathbf{U}}_{k'} \mathbf{H}_{kk'}^H \big) \overleftarrow{\mathbf{V}}_k \\ &= \sum_{k=1}^{K} \overleftarrow{\mathbf{U}}_k \Big(\frac{P}{d\gamma_T} \mathbf{H}_{kk}^H \overleftarrow{\mathbf{V}}_k \overleftarrow{\mathbf{V}}_k^H \mathbf{H}_{kk} \Big) \overleftarrow{\mathbf{U}}_k^H - \sum_{k=1}^{K} \sum_{k' \neq k} \frac{P}{d} \overleftarrow{\mathbf{U}}_{k'} \big(\mathbf{H}_{kk'}^H \overleftarrow{\mathbf{V}}_k \overleftarrow{\mathbf{V}}_k^H \mathbf{H}_{kk'} \big) \overleftarrow{\mathbf{U}}_{k'}^H \\ &= \sum_{k=1}^{K} \overleftarrow{\mathbf{U}}_k \Big(\frac{P}{d\gamma_T} \overleftarrow{\mathbf{H}}_{kk} \overleftarrow{\mathbf{V}}_k \overleftarrow{\mathbf{V}}_k^H \overleftarrow{\mathbf{H}}_{kk}^H \Big) \overleftarrow{\mathbf{U}}_k^H - \sum_{k'=1}^{K} \sum_{k \neq k'} \frac{P}{d} \overleftarrow{\mathbf{U}}_{k'} \big(\overleftarrow{\mathbf{H}}_{k'k} \overleftarrow{\mathbf{V}}_k \overleftarrow{\mathbf{V}}_k^H \overleftarrow{\mathbf{H}}_{k'k}^H \big) \overleftarrow{\mathbf{U}}_{k'}^H . \end{aligned}$$

This yields that given fixed values of $\overleftarrow{\mathbf{V}}_1, \dots, \overleftarrow{\mathbf{V}}_K$, we have

$$\max_{\overleftarrow{\mathbf{U}}_1, \dots, \overleftarrow{\mathbf{U}}_K} T = \sum_{k=1}^{K} \max_{\overleftarrow{\mathbf{U}}_k} \overleftarrow{T}_k, \quad (20)$$

with

$$\overleftarrow{T}_k = \overleftarrow{\mathbf{U}}_k^H \Big(\frac{1}{\gamma_T} \frac{P}{d} \overleftarrow{\mathbf{H}}_{kk}^H \overleftarrow{\mathbf{V}}_k \overleftarrow{\mathbf{V}}_k^H \overleftarrow{\mathbf{H}}_{kk}^H - \sum_{k' \neq k} \frac{P}{d} \overleftarrow{\mathbf{H}}_{kk'} \overleftarrow{\mathbf{V}}_{k'} \overleftarrow{\mathbf{V}}_{k'}^H \overleftarrow{\mathbf{H}}_{kk'}^H \Big) \overleftarrow{\mathbf{U}}_k^H . \quad (21)$$

As such, given fixed values of $\mathbf{U}_1, \dots, \mathbf{U}_K$, the computed $\mathbf{V}_1, \dots, \mathbf{V}_K$ at every iteration maximize T. To conclude, T is an increasing function at every iteration and it remains to show that it is bounded in order to prove its convergence.

Assume that T is not bounded, therefore $\exists k$ so that $T_k \to +\infty$ which means that $\forall M > 0\ \exists \mathbf{V}_1, \dots, \mathbf{V}_K$ and $\mathbf{U}_1, \dots, \mathbf{U}_K$ so that

$$\mathbf{U}_k\left(\frac{1}{\gamma_T}\frac{P}{d}\mathbf{H}_{kk}\mathbf{V}_k\mathbf{V}_k^H\mathbf{H}_{kk}^H - \sum_{k'\neq k}\frac{P}{d}\mathbf{H}_{kk'}\mathbf{V}_{k'}\mathbf{V}_{k'}^H\mathbf{H}_{kk'}^H\right)\mathbf{U}_k^H > M, \quad (22)$$

leading to the following inequality

$$\frac{\mathbf{U}_k\mathbf{H}_{kk}\mathbf{V}_k\mathbf{V}_k^H\mathbf{H}_{kk}^H\mathbf{U}_k\frac{P}{d}}{M + \mathbf{U}_k\sum_{k'\neq k}\mathbf{H}_{kk'}\mathbf{V}_{k'}\mathbf{V}_{k'}^H\mathbf{H}_{kk'}^H\mathbf{U}_k^H\frac{P}{d}} > \gamma_T. \quad (23)$$

As such, when $M \to +\infty$ we get $0 > \gamma_T$ which is impossible. To conclude, if γ_T can be reached by all the streams of all the links, T is bounded. Finally, T is an increasing and bounded function, therefore, it converges and as such, the proposed iterative algorithm converges.

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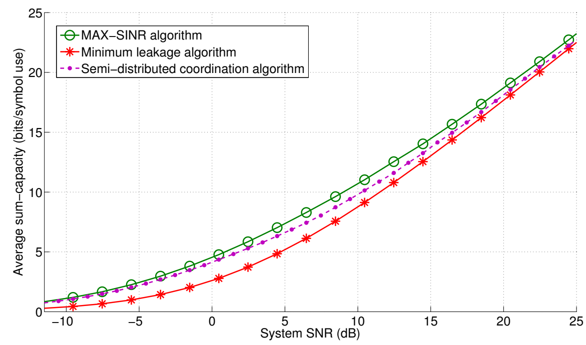

Fig. 1. Average sum-capacity of the proposed coordination algorithms compared to the minimum leakage and the MAX-SINR algorithms when using 1dB target SNR granularity for the 3-user 2×2 MIMO IC with $d_k = 1,\ \forall k = 1, 2, 3$

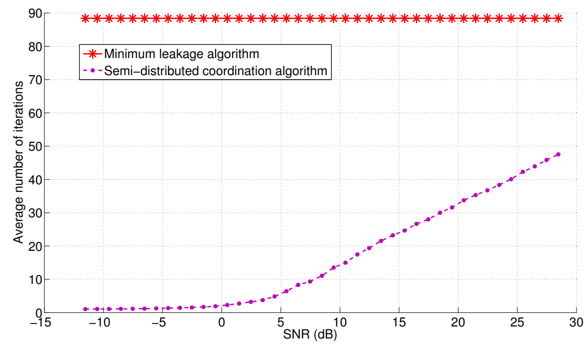

Fig. 2. Average Number of iterations for the proposed coordination algorithms compared to the minimum leakage algorithm for the 3-user 2×2 MIMO IC with $d_k = 1,\ \forall k = 1, 2, 3$