Overview of Faster-Than-Nyquist for Future Mobile Communication Systems

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***Abstract*—Mobile communications has become one of the most developed technologies in the last two decades. Strong demand to increase system capacity is still growing dramatically and non-orthogonal transmission schemes are being considered as a potential solution to improve spectral-power efficiency. The non-orthogonal transmission scheme called Faster-Than-Nyquist (FTN) signaling is surveyed in this paper. FTN is analyzed in both time and frequency domains to show the reason behind its higher capacity as compared to the Nyquist case.**

I. Introduction

In recent years, 4G (sometimes called 3.9G), also known as Long-Term Evolution (LTE) or Evolved UTRA (EUTRA), was released and the commercial service has started in some countries. In LTE, the peak data rate in the downlink and the uplink has increased to 300 Mbps and 75 Mbps, respectively [1] , [2]. LTE realizes very low transmission latency of less than 5 msec within the radio access network and uses bandwidths from 1.4 MHz to 20 MHz. LTE supports only packet transmission mode, and all the data and voice services are also realized in the packet domain. Intra-orthogonal radio access schemes are used and Orthogonal Frequency Division Multiple Access (OFDMA) was adopted for the downlink while Single Carrier (SC)-FDMA, also called Discrete Fourier Transform (DFT)-Spread OFDMA, is used for the uplink. Multiple-Input and Multiple Output (MIMO) transmission is one of the key features to achieve high peak data rate.

Following the LTE Release 8, standardization activity of LTE-Advanced systems began in 2008 to meet the requirements for the user data rate and system capacity proposed by the International Telecommunication Union-Radio communication sector (ITU-R). LTE-Advanced has been developed based on the original LTE radio interface, and various technologies to increase the data rate and spectrum efficiency have been specified such as carrier aggregation (CA), enhanced downlink and uplink MIMO, inter-cell interference coordination in homogeneous and heterogeneous networks, relaying, etc.

Although LTE/LTE-Advanced can accommodate the rapid increase of wireless user access for some years, strong demand to increase the system capacity is still growing dramatically. It is estimated that the total traffic of wireless access will increase beyond 500-fold in 2020 as compared to 2010 [3]. Considering the recent increase use of smart phones and tablets, further development of new radio access techniques is indispensable to enhance the system capacity as well as user data rate for future mobile communication systems beyond LTE-Advanced. The current LTE-Advanced can not accommodate such high traffic without sacrifying user experience which can emerge around 2020. Among the radio access techniques, the transmission signal waveform, multiple access techniques, multiplexing of data, control and reference signals are the representative core techniques in the mobile communication systems although it is becoming more challenging to improve the performance. A potenial solution for this challenge is the non-orthogonal transmission schemes for their improvement of spectral-power efficiency theortically compared to orthogonal transmission schemes. The non-orthogonal transmission scheme Faster-Than-Nyquist (FTN) is one of the approaches being considered for future systems that could improve the spectrum efficiency by increasing the data rate. We will focus in this paper on the FTN concept overview and analysis in the time and frequency domains.

This paper is organized as follows: Section II describes the Faster-Than-Nyquist's concept and the state of the art, Section III analyzes the FTN in the frequency domain showing how we can achieve higher capacities as compared to the Nyquist case, and Section IV describes the system model and gives numerical results.

II. FTN Background

Inter-symbol interference (ISI) is a distortion that occurs to the sent symbols when they overlap partially or totally leading to a degraded detection performance at the receiver. Nyquist's criterion for zero ISI [4], for the samples $x(nT)$ of a signal $x(t)$ is $x(nT) = 1$ for $n = 0$ and $x(nT) = 0$ otherwise where $1/T$ is the symbol (or baud) rate. The FTN concept was introduced by Mazo in 1975 [5] where the signal is modulated faster than the usual rate which introduces intentional ISI at the transmitter side. Fig. 1 shows how the orthogonal transmission of symbols has become non-orthogonal by using FTN. In the Nyquist case, a signal is sent every T seconds while in the FTN case, the signal is sent every τT seconds where $\tau < 1$. Mazo showed that sending sinc pulses up to 25% faster doesn't decrease the minimum Euclidean distance between symbols for an uncoded system using binary modulation. The complexity of FTN lies in the receiver side which is

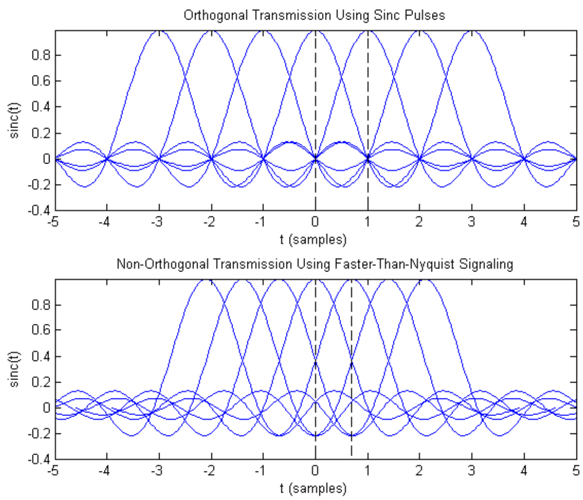

Fig. 1. Nyquist vs. FTN signaling in time domain

responsible for compensating the intentional ISI introduced at the transmitter.

In 2003, Liveris and Georghiades [6] investigated the structure of error events for binary FTN signaling and used constrained coding together with soft interference-cancellation techniques combined with turbo equalization to extend the theoretical throughput gains and/or performance gains to practical ones. They used root raised cosine pulses in their simulation results. In 2006, Rusek and Anderson [7] investigated the lower and upper bounds of the information rates using root raised cosine pulses for binary, quaternary and octal FTN schemes. They proved that in many cases, FTN gives higher information rate than the Nyquist case due to the benefit of using the *excess* pulse bandwidth, which is the small bandwidth added by a pulse that decays slower than a sinc pulse and satisfies the Nyquist criterion. In 2008, non-binary FTN was investigated in detail [8] describing a method to compute the minimum distance and testing it with the M-algorithm receiver. BCJR algorithm [9] is a maximum aposteriori decoding algorithm and the M-algorithm BCJR (M-BCJR) is a reduced states version of the BCJR. In 2010, three different BCJR algorithms [10]; backup M-BCJR, simple M-BCJR and truncated BCJR algorithms, were investigated in order to be used with FTN systems. The decoding at the receiver side was done in an iterative manner using Turbo equalization. The Backup M-BCJR shows a better BER performance compared to the two other algorithms. McGuire and Sima [11] introduced a way of implementing discrete block FTN signaling using matrix multiplication which would reduce the receiver complexity in case of AWGN channels.

Rusek and Anderson also investigated the constrained capacities for FTN [12] and compared their capacity computations with signals using orthogonal modulation. They showed that FTN achieve higher capacities when using non-sinc pulse shapes due to their *excess* pulse bandwidth. In case of using sinc pulses, FTN signals achieve the same capacity as orthogonal signals. More discussion on capacities will be given in the following section. Kim and Bajcsy in [13] analyzed the information rates and their upper bounds for cyclostationary FTN signaling using AWGN and continuous time ISI channels. They showed that FTN achieves higher capacities on ISI channels using the waterfilling technique compared to normal iid transmitted symbols.

The idea of FTN was extended from single carrier to multicarrier in [14] where Mazo's limit, which is the symbol time of $0.802T$, is applied over two dimensions, time and frequency. It was shown that this limit decreases when it is applied over two dimensions as compared to the one dimension, the time domain only, so that higher data rate is achieved even when using a small number of subcarriers. In [15], the transmitter and receiver models for the two dimensional FTN were investigated. Isotropic Orthogonal Transform Algorithm (IOTA) filters were used for pulse shaping; these pulses are compact in both time and frequency similar to Gaussian pulses and achieve a reasonable complexity for the transmitter architecture in case of multicarrier FTN. More information on IOTA pulses is found in [16]. A look-up table with precalculated coefficients was used for FTN mapping and the receiver was based on iterative decoding together with successive interference cancellation. Multicarrier FTN can be applied easily to OFDM systems.

In 2012, Zhou, Li and Wang [17] investigated the generalized FTN signaling where arbitrary modulation pulses were used. They investigated the design of the pulses which not only reduce the detection complexity but also achieve the channel capacity.

III. FTN Capacity overview

In this section, the capacity of FTN signaling is presented by analyzing the FTN signal in the frequency domain. Nyquist criterion for ISI avoidance [4] states that if $X(f)$, which is the Fourier transform of the transmitted signal $x(t)$, satisfies (1) then we obtain ISI free received signal.

$$\frac{1}{T}\sum_{k=-\infty}^{\infty} X(f+\frac{k}{T}) = 1 \text{ for all } f. \tag{1}$$

The minimum bandwidth of a pulse satisfying the Nyquist criterion is $1/T$ for $|f| \leqslant 1/2T$, normalizing the amplitude to 1, for a sinc pulse. The sinc pulse is not used in practice because it decays too fast in the frequency domain and its impulse response is infinite in the time domain. We need a pulse decaying slower and thus having an *excess* pulse bandwidth and can still satisfy Nyquist criterion. We chose a raised cosine pulse for the analysis.

Now we compare sending raised cosine pulses at the Nyquist rate $1/T$ and at the FTN rate $1/\tau T$. For the first case, Fig. 2 shows that according to the Nyquist criterion, the amplitude of the pulses adds up to 1. The original signal is fully reconstructed as a rectangular pulse limited from $-1/2T$ to $1/2T$ after applying an anti-aliasing low pass filter at the receiver side which means that no information is lost. For the

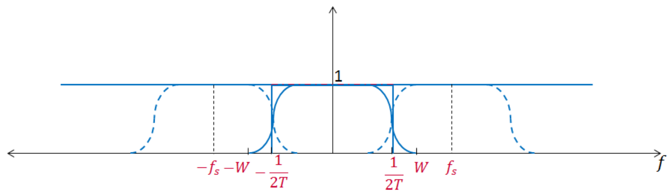

Fig. 2. Transmission of raised cosine pulses at Nyquist rate

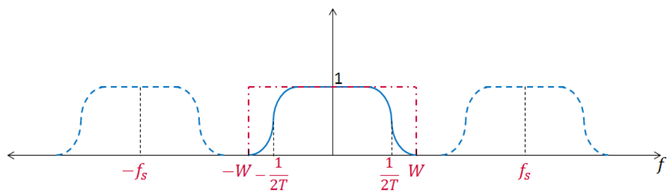

Fig. 3. Transmission of raised cosine pulses at FTN rate

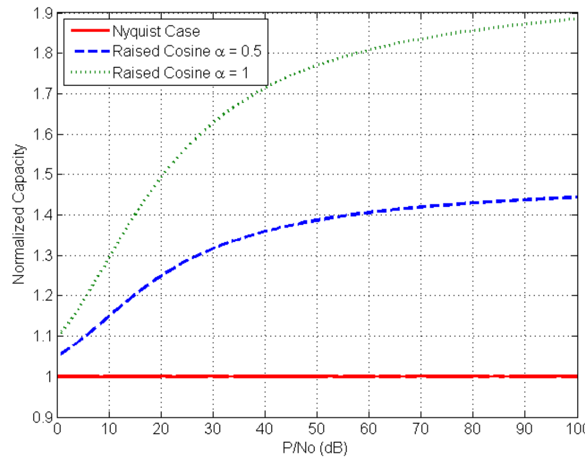

Fig. 4. Normalized Capacity for Nyquist and FTN cases

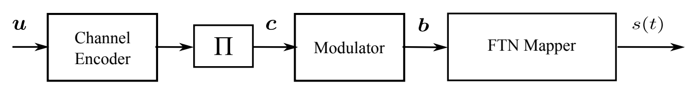

Fig. 5. Transmitter main blocks of the FTN system

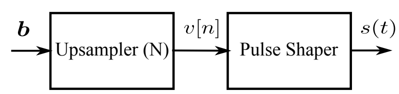

Fig. 6. FTN Mapper

second case, applying FTN in the time domain means that the pulses at the frequency domain are shifted further apart. To reconstruct the original signal at the receiver side, a low pass filter is applied that has a bandwidth W which is larger than $1/T$. See Fig. 3, the entire bandwidth of the received pulse is included in the low pass filter.

Generally Shannon defined the capacity of an AWGN channel as:

$$C = \int_0^W \log_2 \left[1 + \frac{2P}{N_0} |H(f)|^2 \right] df,$$

where W is the one sided bandwidth of the signal, P is the average power of the signal, N_0 is the white noise spectral density and $h(t)$ is the pulse signal of unit energy such that $H(f)$ is the Fourier transform of $h(t)$.

In [12], it was shown that the FTN capacity of a Gaussian alphabet for bandlimited pulses to W Hertz is computed using

$$C_{FTN} = \int_0^W \log_2 \left[1 + \frac{2P}{N_0} |H(f)|^2 \right] df \tag{2}$$

for optimal $\tau = 1/2WT$ that prevents the aliasing effect while the Nyquist capacity is computed using

$$C_N = \int_0^{1/2T} \log_2 \left[1 + \frac{2PT}{N_0} \right] df. \tag{3}$$

We can observe the reason that the FTN capacity is larger than the Nyquist capacity because of the *excess* pulse bandwidth. After evaluating equations (2) and (3), the result is shown in Fig. 4, normalized to the Nyquist capacity when the roll-off factors of the raised cosine pulse are $\alpha = 0.5$ and 1.0. The figure indicates that the asymptotic gain tends to be equal to the *excess* bandwidth for high SNR.

IV. FTN SYSTEM MODEL

In this section, we will investigate a basic model for implementing the FTN concept. In Fig. 5, the main blocks of the transmitter are shown where $\boldsymbol{u}$ is the uncoded signal, $\boldsymbol{c}$ is the channel encoded signal and $\boldsymbol{b}$ is the modulated signal where M-ary modulation schemes are used and $s(t)$ is the transmitted signal defined by (4). One possible way to implement the FTN mapper is to use an upsampler of factor N and a pulse shaper $h(t)$ which is symmetric, has a unit energy $\int_{-\infty}^{\infty} |h(t)|^2 dt = 1$ and upsampled by factor M where $N < M$. The FTN rate is equal to M/N; $\tau = N/M < 1$. The intentional ISI effect generated by the FTN signaling is obtained from the different values of the upsampling factors. Fig. 6 shows the block diagram of the FTN mapper, the upsampled signal $v[n]$ is defined by equation (5).

$$s(t) = \sum_{k=0}^{\infty} b[k] h(t - k\tau T). \tag{4}$$

$$v[n] = \begin{cases} b\left(\frac{n}{N}\right) & n = 0, N, 2N, ... \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

In our model, the signal is transmitted through an AWGN channel such that the received signal $y(t)$ is given by

$$y(t) = s(t) + w(t),$$

where $w(t)$ is white noise of zero mean and two sided power spectral density $N_0/2$.

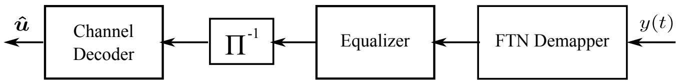

Fig. 7. Receiver main blocks of the FTN system

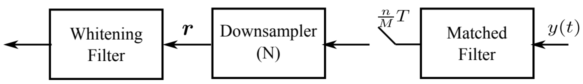

Fig. 8. FTN Demapper

The receiver main blocks are shown in Fig. 7 and they consist of an FTN demapper responsible for returning the received signal to the symbol rate, an equalizer responsible for eliminating the ISI resulting from the FTN and then generating soft output bits fed into the channel decoder. The received signal $y(t)$ passess through the FTN demapper and the resulting signal $r(t)$ defined by (6) is then sampled every τT. Similar to the proposed model of the FTN mapper at the transmitter side, the FTN demapper first applies a matched filter $h^*(-t)$ on $y(t)$, having the same pulse shape used at the transmitter side and upsampled by a factor M. The resulting signal $r(t)$ is then downsampled by a factor N. Finally a whitening filter is applied to de-color the noise. Fig. 8 illustrates the procedure.

$$\begin{aligned} r(t) &= y(t) * h^*(-t) \\ &= (s(t) + w(t)) * h^*(-t) \\ &= \left(\sum_{k=0}^{\infty} b[k] h(t - k\tau T) \right) * h^*(-t) + \tilde{w}(t) \end{aligned} \tag{6}$$

where $\tilde{w}(t)$ is colored noise.

To evaluate our system model, we perform numerical simulations. The simulation parameters are: (7,5) convolutional code of rate = 1/2, root raised cosine pulse shape having a roll-off factor $\alpha = 0.5$, BPSK modulation, soft output BCJR equalizer and the number of information bits is 1000 per frame having 100 frames. The average bit error rate (BER) curves are shown in Fig. 9. The BER curves for the FTN cases of $\tau = 0.9$ and 0.8 are very close to the Nyquist case. As τ increases, the BER increases.

V. Conclusion

The Faster-Than-Nyquist's (FTN) concept in both time and frequency domains was discussed showing its main advantages of higher capacity gain and higher data rate compared to the Nyquist case. This capacity gain in FTN is obtained only if non-sinc pulses are used that have *excess* pulse bandwidth while the higher data rate comes from the fact that we are sending more than one symbol at a unit time.

Some of the technical issues that we would consider in the future are applying an appropriate reduced state equalizer, as the complexity of the equalizer increases drastically for higher order modulation schemes, together with different channel codes and examining the gain for practical systems.

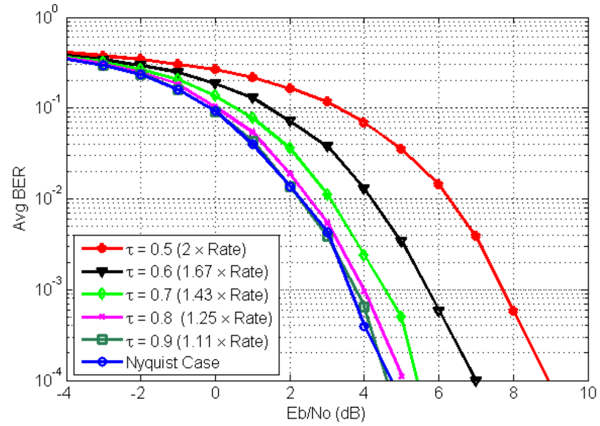

Fig. 9. Average BER curves for an FTN system

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