Sum-Rate Analysis for Full-Duplex Underlay Device-to-Device Networks

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***Abstract*—A theoretical framework is presented for the evaluation of sum ergodic rate of a full-duplex underlay device-to-device networks. The sum-rate of the full-duplex network is compared with a half-duplex network with equivalent radio frequency hardware complexity. Closed-form expressions are derived for the sum ergodic rate of the systems. Furthermore, the sum-rate performances are investigated for the case when a transmit power constraint is imposed on the underlay network to minimize the interference on the cellular network. The analytical results presented can be used as a tool to identify when full-duplex transmissions are viable in underlay device-to-device networks.**

***Index Terms*—Full-duplex transmission, ergodic rate, self-interference.**

I. Introduction

Almost all the current wireless communications technologies enable bidirectional communications using frequency division duplexing (FDD) or time division duplexing (TDD). This requires allocating orthogonal time (TDD) or spectral (FDD) resources for transmission (Tx) and reception (Rx). Although this mechanism has proved to be extremely successful, it may not be able to cater for the spectral efficiency requirements of future generation wireless communication technologies. Therefore, recently there has been a surge of interest on the systems where Tx and Rx is performed using the same time or spectral resources. These systems are commonly known as full-duplex (FD) systems, while conventional TDD and FDD systems are referred to as half-duplex (HD) systems. An inherent challenge of FD system is the interference on Rx by it's own Tx. This is known as self interference (SI). Recent studies [1] have proposed SI cancelation schemes that can achieve 90 dB isolation between the Tx and the Rx. Experimental results of [2] have shown that FD systems are capable of achieving higher spectral efficiencies than HD systems for SI isolations above 74 dB. However, these gains have been observed in point-to-point FD systems with short distance between the nodes. Exploiting this fact, we aim to investigate the applicability of FD technique in device-to-device (D2D) networks, where the communications are generally short range.

In a D2D network, users connect with each other without communicating through the central base station (BS) to improve the overall spectral efficiency of the system, to reduce battery consumption and to reduce the workload of the BS [3], [4]. Recently the idea of underlaying D2D communications has gained interest in cellular networks, where D2D network coexists with a conventional cellular network while maintaining a maximum interference constraint on the cellular network. The results of [5] point out the feasibility of underlay D2D networks in 3rd generation partnership project (3GPP) long term evolution-Advanced (LTE-A) networks.

Surveys [6] indicate that the D2D type communications will become more commonplace in the future. It has been added as a study item in 3GPP and investigated as a feature in possible 5th generation (5G) communications. Several works have proposed efficient communication techniques for D2D networks including resource allocation [7], and power optimization [8]. However, all the previous works on D2D networks, did not consider FD communications for D2D users. It is interesting to combine the concepts of FD and D2D, since it may allow us to harvest the benefits of both technologies to improve the spectral efficiency of wireless communications. For example, latest wireless standards such as LTE-A do not support FD communications. However, introduction of FD communications to underlay D2D users that coexist with other LTE-A users, may result in increased total network throughput without changing the infrastructure of the network.

In this paper, we investigate the feasibility of FD communications in underlay D2D networks. We study the sum ergodic rate of an underlay D2D network when D2D users operate in FD mode and compare the performance to a HD underlay D2D network with equivalent total energy and radio frequency (RF) hardware complexity. Furthermore, we compare the performance of both HD and FD D2D networks to a conventional cellular network. Next, we analyze the case when transmit power adaptation is used at the D2D nodes to maintain a maximum interference constraint on the cellular network. Our analytical results can be used to identify when FD networks are advantageous to improve the overall system sum-rate. To the best of authors' knowledge, this is the first work investigating the sum ergodic rate of FD D2D networks.

The remainder of this paper is organized as follows. In Section II, we present FD, and HD D2D network models. Section III presents the ergodic rate analysis of each network configuration. Section IV presents numerical results and comparisons while Section V concludes this paper. The derivations are presented in appendices.

II. System Model

In this Section, we present the network models used for our analytical study. We consider a single circular cell with radius R_c with BS located at the center of the cell. A single cellular user (CU) and a pair of D2D users (D1 and D2) are located

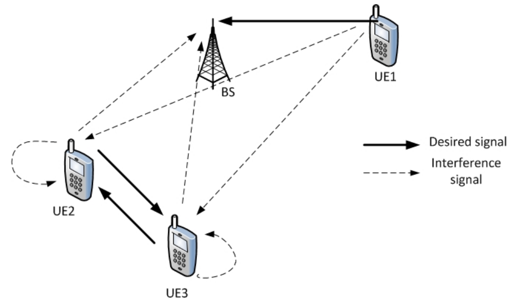

Fig. 1. Underlay D2D network model, where the solid lines denote the desired signals and the dashed lines denote the interference links.

inside the cell. The D2D pair is assumed to be operating using the same resources as the uplink of the CU. We assume that the transmitters do not have channel state information (CSI). Two communication modes are considered for the D2D pair.

1) **FD Mode**: In the FD mode, both D2D users transmit and receive at the same time instant using the same frequency band. We assume a 1×1 FD D2D system where each node requires 1 up-converting RF chain for Tx, 1 down-converting RF chain for Rx and 1 up-converting RF chain for SI cancelation. A total transmission period of T seconds is considered. The available power at D2D nodes is P_d. Therefore, the total energy consumed by the D2D pair in the FD mode is $2P_dT$.
2) **HD Mode**: In the HD mode, each D2D user transmit for $\frac{T}{2}$ time period. In order to make a fair comparison between the FD mode and the HD mode, we assume that both systems have equal RF hardware complexity. Since in the FD mode, a D2D node uses 2 up-converting RF chains and 1 down-converting RF chain, similar to [2], [9], we define an equivalent HD system with two up-converting RF chains for Tx and one down-converting RF chain for Rx. Therefore, the RF equivalent HD system for 1×1 FD pair is a 2×1 multiple-input single-output (MISO) system. In order to keep the total energy consumption of the two modes equal, each antenna transmits with power P_d with unit energy symbols, such that the total energy consumed is $2P_dT$.

In the first stage of this work, we assume that there is no interference coordination between the cellular network and the D2D pair. The channels between all the entities are assumed to be flat Rayleigh faded with average fading power of unity. We assume a log-distance path loss model with reference distance of 10 m. The performance metric we are interested in is the sum ergodic rate of the network. We derive closed-form expressions for the sum ergodic rate for both FD and HD modes. Furthermore, to understand the feasibility of D2D networks over conventional networks, we compare both FD and HD D2D systems with a two-way relay network (TWRN), where the BS acts as a relay between the two D2D users to facilitate the data exchange in a spectrally efficient manner.

III. Ergodic Rate Analysis

In this Section, we present the sum ergodic rate analysis for each D2D communication mode.

A. FD Mode

The signal received at the BS can be written as

$$y_{BS,\text{FD}} = \sqrt{P_u L_u} h_u s_u + \sum_{k=1}^{2} \sqrt{P_d L_{dk}} h_{dk} s_{dk} + n_1$$

where P_u is the transmit power of CU, L_u is the path loss between the CU and the BS, h_u is the flat fading channel coefficient between the CU and the BS, s_u is the unit energy signal transmitted by the CU, n_1 is the additive white Gaussian noise (AWGN) with variance N_0 at the BS, s_{dk} is the unit energy signal transmitted by the k^{th} D2D user, L_{dk} and h_{dk} are the path loss and the flat fading channel coefficient between the k^{th} D2D user and the BS, respectively. The signal-to-interference-plus-noise ratio (SINR) at the BS can be written as

$$\gamma_{U,\text{FD}} = \frac{P_u L_u |h_u|^2}{\sum_{k=1}^{2} P_d L_{dk} |h_{dk}|^2 + N_0}. \quad (1)$$

The path loss coefficients are computed using

$$L(dB) = \begin{cases} 32 + 20\log_{10}(f_c d) & \text{if } d \le 10 \\ 60 + 10\eta \log_{10}(d/10) & \text{if } d > 10 \end{cases} \quad (2)$$

where $f_c = 2.4$ is the career frequency in GHz, d is the distance between the nodes and η is the path loss exponent. The received signal at the k^{th} D2D user is given by

$$y_{k,\text{FD}} = \sqrt{P_d L_{12}} h_{kj} s_j + \sqrt{P_u L_{uk}} h_{uk} s_u + I_k + n_{kd}, \quad k, j \in \{1,2\}, k \neq j \quad (3)$$

where L_{uk}, h_{uk} are the path loss and the fading channel coefficient between the CU and the k^{th} D2D user, L_{12}, and h_{kj} are the path loss and the channel coefficient between the D2D pair, and n_{kd} is the AWGN with variance N_0 at the k^{th} D2D user. The SI I_k at the k^{th} D2D user is modeled as additional Gaussian noise with variance σ_{Ik}^2. The variance σ_{Ik}^2 depends on the SI cancelation technique used at the nodes. According to the experimentally verified SI model given in [2], σ_{Ik}^2 can be computed using

$$\sigma_{Ik}^2(dBm) = P_d(dBm) - L_{SI}(dB) - (\lambda(dB/dBm)(P_d(dBm) - L_{SI}(dB)) + \beta(dB)) \quad (4)$$

where L_{SI} is the passive SI cancelation due to antenna isolation, λ and β are coefficients depending on the active cancelation [10]. The SINR at the k^{th} D2D user can be given as

$$\gamma_{k,\text{FD}} = \frac{P_d L_{12} |h_{kj}|^2}{P_u L_{uk} |h_{uk}|^2 + \sigma_{Ik}^2 + N_0}. \quad (5)$$

The sum ergodic rate of the system can be computed using

$$R_{FD,D2D} = \overbrace{\mathbb{E}\left[\log_2\left(1 + \gamma_{U,\text{FD}}\right)\right]}^{R_{U,\text{FD}}} + \underbrace{\sum_{k=1}^{2} \mathbb{E}\left[\log_2\left(1 + \gamma_{k,\text{FD}}\right)\right]}_{R_{D2D,\text{FD}}} \quad (6)$$

where $R_{U,\text{FD}}$ and $R_{D2D,\text{FD}}$ are the ergodic rates of the CU and D2D pair, respectively. For our analysis, we assume that the position of the CU is randomly located at a distance r_1 from the BS and the D2D pair is fixed at a distance of r_2 from the BS (see Fig. 5). The mean distance between the CU and the k^{th} D2D user can be found using

$$\bar{d}_{uk} = \frac{2\sqrt{A+B}}{\pi}\boldsymbol{K}\left(\sqrt{\frac{2B}{A+B}}\right) \tag{7}$$

where $A = r_1^2 + r_2^2$, $B = 2r_1r_2$, and $\boldsymbol{K}(\cdot)$ is the complete elliptic integral of the second kind [11, 8.112]. The derivation of (7) is given in Appendix A. When calculating the values of L_{uk}, we substitute $\bar{d}_{uk}$ in (2). The ergodic rate of the CU, $R_{U,\text{FD}}$, can be computed by solving the integral

$$R_{U,\text{FD}} = \log_2(e)\int_0^\infty \frac{1-F_{\gamma_{U,\text{FD}}}(x)}{(1+x)}dx \tag{8}$$

where $F_{\gamma_{U,\text{FD}}}(x)$ is the cumulative distribution function (CDF) of $\gamma_{U,\text{FD}}$ given by

$$F_{\gamma_{U,\text{FD}}}(x) = 1-\exp\left(-\frac{x}{\bar{\gamma}_u}\right)\frac{1}{\left(1+\frac{\bar{\gamma}_b}{\bar{\gamma}_u}x\right)^2} \tag{9}$$

where $\bar{\gamma}_u = \frac{P_uL_u}{N_0}$, and $\bar{\gamma}_b = \frac{P_dL_{dk}}{N_0}$. The closed-form solution for (8) is given by

$$\begin{aligned} R_{U,\text{FD}} = \log_2(e)\Upsilon^2 &\left[\frac{1}{1-\Upsilon}\left(\frac{1}{\Upsilon}-\frac{1}{\bar{\gamma}_u}e^{\frac{1}{\bar{\gamma}_b}}\boldsymbol{E}_1\left(\frac{1}{\bar{\gamma}_b}\right)\right)\right. \\ &\left. -\frac{1}{(1-\Upsilon)^2}e^{\frac{1}{\bar{\gamma}_b}}\boldsymbol{E}_1\left(\frac{1}{\bar{\gamma}_b}\right)+\frac{1}{(\Upsilon-1)^2}e^{\frac{1}{\bar{\gamma}_u}}\boldsymbol{E}_1\left(\frac{1}{\bar{\gamma}_u}\right)\right] \end{aligned} \tag{10}$$

where $\Upsilon = \frac{\bar{\gamma}_u}{\bar{\gamma}_b}$, and $\boldsymbol{E}_1(\cdot)$ is the exponential integral function.

The CDF of $\gamma_{k,\text{FD}}$ is given by

$$F_{\gamma_{k,\text{FD}}}(x) = 1-\frac{\Phi\exp\left(-\frac{x(1+\bar{I}_k)}{\bar{\gamma}_{k,\text{FD}}}\right)}{(x+\Phi)} \tag{11}$$

and the ergodic rate of the k^{th} D2D user can be found as

$$R_{D2D,\text{FD}} = \log_2(e)\Phi\left[\frac{e^{\frac{1+\bar{I}_k}{\bar{\gamma}_{k,\text{FD}}}}\boldsymbol{E}_1\left(\frac{1+\bar{I}_k}{\bar{\gamma}_{k,\text{FD}}}\right)}{\Phi-1}+\frac{e^{\frac{1+\bar{I}_k}{\bar{\gamma}_{k,u}}}\boldsymbol{E}_1\left(\frac{1+\bar{I}_k}{\bar{\gamma}_{k,u}}\right)}{1-\Phi}\right] \tag{12}$$

where $\Phi = \frac{\bar{\gamma}_{k,\text{FD}}}{\bar{\gamma}_{k,u}}$, $\bar{\gamma}_{k,\text{FD}} = \frac{P_dL_{12}}{N_0}$, $\bar{\gamma}_{k,u} = \frac{P_uL_{uk}}{N_0}$ and $\bar{I}_k = \frac{\sigma_{Ik}^2}{N_0}$. The derivations of (9), (8), (11) and (12) are shown in Appendix B.

B. HD Mode

In the HD mode, the D2D pair forms a 2×1 MISO system. Therefore, during the first $T/2$ period, 1st D2D user transmits using 2 antennas and the other user receives with a single antenna. Since CSI is not available at the transmitter, D2D users transmit using Alamouti space-time code (STC) to achieve transmit diversity. Then, the received signal at the BS in a particular symbol period during the first $\frac{T}{2}$ interval can be written as

$$y_{T/2,\text{HD}} = \sqrt{P_uL_u}h_us_u + \sum_{k=1}^{2}\sqrt{P_dL_{k,1}}h_{k,1}s_{k,1}+n_1 \tag{13}$$

where $s_{k,1}$ is the transmit signal from the k^{th} Tx antenna of the 1st D2D user, $L_{k,1}$ and $h_{k,1}$ are the path loss and the fading channel between the BS and the k^{th} Tx antenna of the 1st D2D user, respectively. The SINR in the first $T/2$ period is given by

$$\gamma_{T/2,\text{HD}} = \frac{P_uL_u|h_u|^2}{\sum_{k=1}^2 P_dL_{k,1}|h_{k,1}|^2+N_0}. \tag{14}$$

Since the D2D transmitter uses Alamouti STC, we consider two symbol periods for analysis. The received signal vector at the 2nd D2D user can be given as

$$\mathbf{y}_{2,\text{HD}} = \sqrt{P_dL_{12}}\begin{bmatrix} h_1 & -h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} s_{1,1} \\ s_{2,1}\end{bmatrix}+\mathbf{n}+\sqrt{P_uL_{u1}}\mathbf{I}_u$$

where h_1, h_2 are channel gains between the two transmit antennas of the 1st D2D user and the 2nd D2D user, $s_{1,1}$, $s_{2,1}$ are the symbols transmitted by the 1st D2D user, $\mathbf{n}$ is the AWGN vector with covariance matrix $N_0\mathbf{I}$ and $\mathbf{I}_u$ is the interference vector from the CU at the 2nd D2D user. After matched filtering, the SINR per symbol is given by

$$\gamma_{2,\text{HD}} = \frac{P_dL_{12}(|h_1|^2+|h_2|^2)}{P_uL_{u1}|h_{u1}|^2+N_0} \tag{15}$$

The SINRs at the BS and the 1st D2D user in a particular symbol period during the second $T/2$ interval can be found similarly. One can observe that if $L_{k,1} = L_{k,2}\ \forall k\in[1,2]$, the overall SINR at the BS during the time period T in the HD mode has similar form as the SINR in the FD mode. Then, SINR CDFs of the BS are equivalent, and the ergodic rate for the HD mode can be found as (8). Following the procedure given in Appendix B, the CDF of the SINR at each D2D user can be found as

$$\begin{aligned} F_{\gamma_{k,\text{HD}}}(x) = 1-\exp\left(-\frac{x}{\bar{\gamma}}\right)&\left[\frac{\Delta}{x+\Delta}+\frac{x}{\bar{\gamma}_{k,u}(x+\Delta)}\right. \\ &\left.+\frac{x\Delta}{(x+\Delta)^2}\right] \end{aligned} \tag{16}$$

where $\bar{\gamma} = \frac{P_dL_{12}}{N_0}$, and $\Delta = \frac{\bar{\gamma}}{\bar{\gamma}_{k,u}}$. Using the integral identities given in [12], the ergodic rate for each D2D user in the HD mode can be found in closed-form as

$$\begin{aligned} R_{k,\text{HD}} = \frac{\log_2(e)}{2}&\left[\frac{2\Delta e^{\frac{1}{\bar{\gamma}_{k,u}}}\boldsymbol{E}_1\left(\frac{1}{\bar{\gamma}_{k,u}}\right)}{(1-\Delta)}+\frac{\Delta e^{\frac{1}{\bar{\gamma}}}\boldsymbol{E}_1\left(\frac{1}{\bar{\gamma}}\right)}{(\Delta-1)}\right. \\ &-\frac{\Delta}{1-\Delta}-\frac{e^{\frac{1}{\bar{\gamma}}}\boldsymbol{E}_1\left(\frac{1}{\bar{\gamma}}\right)}{\bar{\gamma}_{k,u}(\Delta-1)}+\frac{\Delta^2e^{\frac{1}{\bar{\gamma}_{k,u}}}\boldsymbol{E}_1\left(\frac{1}{\bar{\gamma}_{k,u}}\right)}{(1-\Delta^2)} \\ &\left.-\frac{\Delta e^{\frac{1}{\bar{\gamma}}}\boldsymbol{E}_1\left(\frac{1}{\bar{\gamma}}\right)}{(\Delta-1)^2}\right]. \end{aligned} \tag{17}$$

C. Comparison With TWRN

In order to understand the gain of underlay D2D deployment, we also compare the sum rate results of FD and HD D2D networks with conventional cellular network. To perform this comparison, we select TWRN as the conventional cellular counterpart of the D2D network, since TWRNs have been identified as a spectrally efficient scheme for data exchange between two nodes with the aid of a central network entity.

In the TWRN model, communication occurs in three time slots. In the first time slot, CU communicates with the BS. In the second time slot, the two users (D1 and D2 in D2D model) send their data to the BS which functions essentially as a relay. The BS applies a gain on the received signal and transmits the amplified signal in the third time slot. The users D1 and D2 subtract their own signal parts from the signal received and use the remainder for data decoding. In order to make comparisons fair, we assume D1 and D2 are equipped with 2 Tx antennas and the BS (relay) is equipped with a single antenna. The users apply maximal ratio transmission (MRT) for data transmission and maximal ratio combining (MRC) for reception. A similar system model was analyzed in [13] for users with correlated antennas. For simplicity, we do not consider the antenna correlation in this paper. To make sure that the energy consumptions of D2D model and the TWRN model are equal, per antenna transmit power of D1 and D2 is set to $\frac{P_d}{4}$ and the transmit power of the BS is set to P_d thus making the total energy consumed by the network during the two time slots to be equal to $2P_dT$. The sum ergodic rate for the system can be written as

$$R_{sum,TWRN} = \frac{R_{U,TWRN} + 2R_{TWRN}}{3} \tag{18}$$

where $R_{U,TWRN}$ is the ergodic rate of the CU and R_{TWRN} is the ergodic rate of the TWRN. The ergodic rate of the CU is given by

$$R_{U,TWRN} = \exp\left(\frac{1}{\bar{\gamma}_u}\right)\boldsymbol{E}_1\left(\frac{1}{\bar{\gamma}_u}\right). \tag{19}$$

The ergodic rate of the TWRN can be found using the results presented in [13, Sec. III-C].

D. Impact of an Interference Constraint

Next, we analyze the sum rate of the system when a maximum interference constraint is imposed on the D2D network, when the D2D nodes are operating in the FD mode. In this situation, the BS measures the interference it receives from the D2D users and informs them through a control channel to limit their transmit power accordingly. The maximum interference threshold is computed to maintain a minimum quality of service (QoS) guarantee for the CU. Then the transmit power at the D2D users are adjusted according to

$$P_{k,D2D} = \min\left(\frac{I_{th,k}}{L_{dk}|h_{dk}|^2}, P_d\right) \tag{20}$$

where $I_{th,k}$ is the maximum interference allowed from the k^{th} D2D user. The SINR at the k^{th} D2D user is given by

$$\gamma_{k,D2D} = \frac{P_{k,D2D}L_{12}|h_{kj}|^2}{P_u L_{uk}|h_{uk}|^2 + \sigma_{Ik} + N_0}. \tag{21}$$

In this situation, to compute the statistics of SI, one must require the average transmit power of D2D nodes. The average transmit power $\bar{P}_{k,D2D}$ can be found as

$$\bar{P}_{k,D2D} = \sqrt{\Omega_k P_d}\exp\left(-\frac{\Omega}{2P_d}\right)\boldsymbol{W}_{-\frac{1}{2},0}\left(\frac{\Omega}{P_d}\right) + P_d\left(1 - \boldsymbol{Q}\left(1, \frac{\Omega}{P_d}\right)\right) \tag{22}$$

where $\Omega = \frac{I_{th,k}}{L_{dk}}$, $\boldsymbol{W}_{a,b}(\cdot)$ is the Whittaker function [11, eq. 9.221.1] and $\boldsymbol{Q}(c,d)$ is the incomplete gamma function [11, eq. 8.350.2]. (The derivation of (22) is given in Appendix C.) Then the variance of SI can be computed by replacing P_d with $\bar{P}_{k,D2D}$ in (4).

Obtaining an exact expression for the ergodic rate of D2D users appears to be intractable with the SINR expression (21). Therefore, we approximate the interference component from the CU by its average value $P_u L_{uk}$ and treat interference as an additional Gaussian noise. With this assumption, the CDF of $\gamma_{k,D2D}$ can be found using [14, eq. (8)], and has the form

$$F_{\gamma_{k,D2D}}(x) = 1 + \exp\left(-\frac{L_{12}\sigma^2 x}{P_d}\right)\left(\frac{\exp\left(-\frac{\alpha L_{12} I_{th,k}}{P_d}\right)}{\frac{\alpha I_{th,k}}{\sigma^2 x} + 1} - 1\right)$$

where $\sigma^2 = P_u L_{uk} + \sigma_{Ik} + N_0$ and $\alpha = \frac{L_{dk}}{L_{12}}$. Now the ergodic rate can be derived as

$$\begin{aligned} R_{k,D2D} = {} & \exp\left(\frac{L_{12}\sigma^2}{P_d}\right)\boldsymbol{E}_1\left(\frac{L_{12}\sigma^2}{P_d}\right) - \frac{\exp\left(-\frac{\alpha L_{12} I_{th,k}}{P_d}\right)}{\frac{L_{12}\sigma^2}{P_d}\left(\frac{\alpha I_{th,k}}{\sigma^2} - 1\right)} \\ & + \frac{\exp\left(-\frac{\alpha L_{12} I_{th,k}}{P_d}\right)\exp\left(\frac{L_{12}\sigma^2}{P_d}\right)\boldsymbol{E}_1\left(\frac{L_{12}\sigma^2}{P_d}\right)}{\left(\frac{\alpha I_{th,k}}{\sigma^2} - 1\right)} \\ & + \frac{\frac{\alpha I_{th,k}}{\sigma^2}\boldsymbol{E}_1\left(\frac{\alpha L_{12} I_{th,k}}{P_d}\right)}{\left(1 - \frac{\alpha I_{th,k}}{\sigma^2}\right)} - \frac{\exp\left(-\frac{\alpha L_{12} I_{th,k}}{P_d}\right)}{\frac{L_{12}\sigma^2}{P_d}\left(1 - \frac{\alpha I_{th,k}}{\sigma^2}\right)} \end{aligned} \tag{23}$$

For the HD mode, exact analysis becomes intractable with Alamouti STC transmission. Therefore, we propose to approximate the ergodic rate of the D2D users by substituting $\bar{P}_{k,D2D}$ instead of P_d in (17).

IV. Numerical Results and Discussion

In this Section, we provide some numerical results to verify the analysis conducted in Sec. III. For our numerical results, we assume a circular cell of radius 250 m. The CU location is uniformly distributed at a distance r_1 from the BS. The position of the D2D pair is fixed at a distance r_2 from the BS. Carrier frequency of 2.4 GHz is used for path loss calculations with path loss exponent of 2. The D2D pair is assumed to be located 10 m apart from each other. The transmit power of the CU is set to 24 dBm. Noise variance N_0 is assumed to be -116.4 dBm. In all simulation results, 10000 random CU locations were used with 1000000 independent channel realizations.

Fig. 2 shows the sum rate performance comparisons for each mode as a function of the distance of the D2D pair from the BS. Transmit power of the D2D users is set to 20 dBm with no maximum interference constraint. The CU is located near the

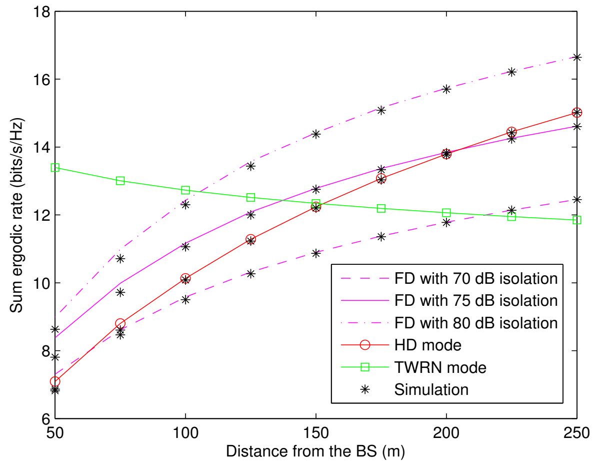

Fig. 2. The sum ergodic rates of the system as a function of the distance of D2D pair from the BS

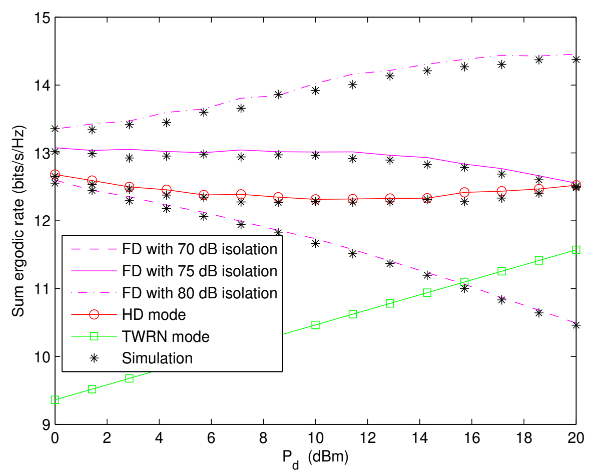

Fig. 3. The sum ergodic rates of the system as a function of the distance of D2D pair from the BS

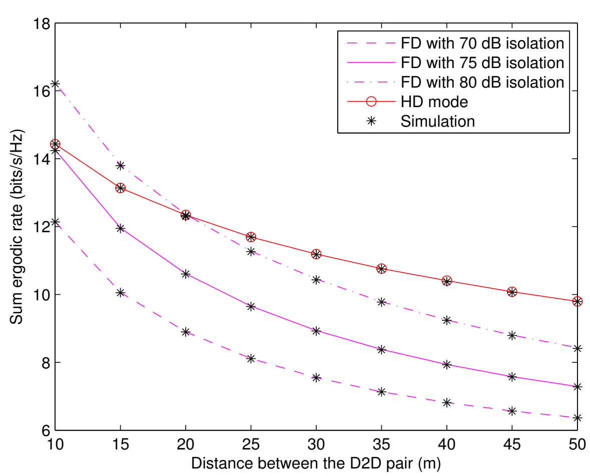

Fig. 4. The sum ergodic rates of the system as a function of the distance of D2D pair from the BS

BS at a distance of 75 m. One can observe that the theoretical results are in excellent agreement with the simulation results. It can be observed that the D2D communication is beneficial when the users are closer to the cell edge. The HD mode outperforms FD mode when the self interference cancellation is below 75 dB.

Fig. 3 shows the system sum rate comparisons for each mode as a function of the transmit power of D2D pair. The sum rate in the FD mode decreases with increasing transmit power due to the increase in SI. The ergodic rate of the HD mode remains almost constant and outperforms FD mode for SI cancellations below 75 dB. When the SI cancellation capability increases, the sum rate tends to improve with the increasing transmit power.

Fig. 4 gives the sum rate performances with the distance between the D2D pair, with $P_d = 20$ dBm. It is clear that the FD mode outperforms the HD mode when the distance between the D2D pair is shorter. As the distance increases, FD with 80 dB SI cancellation results in lower sum rate than the HD mode.

V. Conclusion

A theoretical framework was derived to evaluate the sum ergodic rate of underlay D2D network operating in FD and HD modes. Closed-form expressions were derived for the sum ergodic rates, when the location of the D2D pair is fixed, while the CU is randomly located. The derived expressions can be conveniently evaluated using common mathematical software packages. The theoretical results were verified using extensive Monte-carlo simulations. The new expressions can be used to save computation time in performance evaluation of underlay D2D networks operating in FD and HD modes. Analytical results can be used to identify the performance crossover point between the HD and FD modes.

Appendix A

In this Appendix, we present the derivation of (7). The CU is randomly located at a distance r_1 from the BS and the locations of D1 and D2 are fixed. We assume that the angle θ is a uniformly distributed random variable (RV) in the interval $[-\pi, \pi]$. We consider the case when $r_2 > r_1$ where D2D communication is beneficial over conventional system. The results for $r_2 < r_1$ can be found in a similar manner. Applying the cosine law on the triangle, the squared distance can be found as

$$d_{u,k}^2 = r_1^2 + r_2^2 - 2r_1 r_2 \sin(\theta).$$

Applying standard transformation principles, the probability density function (PDF) of $d_{u,k}^2$ can be found as

$$f_{d_{u,k}^2}(y) = \frac{1}{\sqrt{1-\left(\frac{y-A}{B}\right)^2}}, \quad y \in \{A-B, A+B\}.$$

The mean distance can be computed as

$$\bar{d}_{u,k} = \int_{A-B}^{A+B} \frac{\sqrt{y}}{\sqrt{1-\left(\frac{y-A}{B}\right)^2}} dy.$$

Applying the variable transformation $\frac{y-A}{B} = \cos(x)$ and solving the resulting integral using [11, eq. 3.670.1], the mean distance between the CU and D1 can be found as in (7). Due to the symmetry, the mean distance between CU and D2 is also equal to $\bar{d}_{u,k}$.

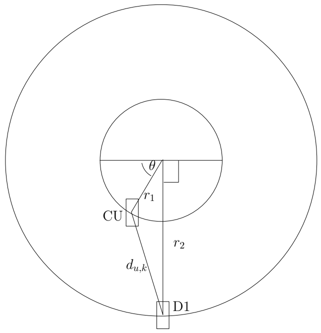

Fig. 5. Distance between CU and D1

APPENDIX B

In this Appendix, we present the derivations of (9), (8), (11) and (12). Assuming $L_{d1} = L_{d2}$, the total interference power at the BS is chi-square distributed and the PDF is given by

$$f_{\gamma_I}(x) = \frac{x}{\bar{\gamma}_b} \exp\left(-\frac{x}{\bar{\gamma}_b}\right).$$

The CDF of $\gamma_{U,\text{FD}}$ is found using

$$\begin{aligned} F_{\gamma_{U,\text{FD}}}(x) &= \Pr\left(\frac{P_u L_u |h_u|^2}{\sum_{k=1}^{2} P_d L_{dk} |h_{dk}|^2 + N_0} \leq x\right) \\ &= \int_0^\infty 1 - \exp\left(-\frac{x(y+1)}{\bar{\gamma}_u}\right) f_{\gamma_I}(y) dy \qquad (24) \end{aligned}$$

and a closed-form solution is found using [11, 3.381.4]. Substituting (9) in (8) and applying the result in [12], the ergodic rate is found in closed-form as (8). The derivation of (11) and (12) follow the same method as (9) and (8), and can be deduced in a straightforward manner.

APPENDIX C

In this Appendix, we present the derivation of (22). The RV $P_{k,D2D}$ is a mixed RV with a continuous and discrete components in the PDF. The RV $X = \frac{\Omega}{|h_{dk}|^2}$ is inverse gamma distributed with PDF

$$f_X(x) = \Omega x^{-2} \exp\left(-\frac{\Omega}{x}\right).$$

Then $\bar{P}_{k,D2D}$ can be computed using

$$\bar{P}_{k,D2D} = \underbrace{\int_0^{P_d} x f_X(x) dx}_{\mathcal{I}_1} + \underbrace{P_d \int_{P_d}^\infty f_X(x) dx}_{\mathcal{I}_2}$$

with

$$\mathcal{I}_2 = P_d \left(1 - \boldsymbol{Q}\left(1, \frac{\Omega}{P_d}\right)\right).$$

Using the variable transformation $\frac{\Omega}{x} = t$ and applying the integral identity [11, eq. 3.381.6] the integral $\mathcal{I}_1$ can be solved in closed-form.

REFERENCES

[1] E. Everett, *Full-Duplex Infrastructure Nodes: Achieving Long Range with Half-duplex Mobiles.* Masters Thesis, Rice University., 2012, available: http://hdl.handle.net/1911/64704.

[2] M. Duarte, C. Dick, and A. Sabharwal, "Experiment-driven characterization of full-duplex wireless systems," *IEEE Trans. Wireless Commun.*, vol. 11, no. 12, pp. 4296–4307, 2012.

[3] K. Doppler, C. H. Yu, C. B. Ribeiro, and P. Janis, "Mode selection for device-to-device communication underlaying an LTE-Advanced network," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, 2010.

[4] B. Kaufman and B. Aazhang, "Cellular networks with an overlaid device to device network," in *Proc. 42nd Asilomar Conference on Signals, Systems and Computers*, 2008, pp. 1537–1541.

[5] K. Doppler, M. Rinne, C. Wijting, C. B. Ribeiro, and K. Hugl, "Device-to-device communication as an underlay to LTE-Advanced networks," *IEEE Commun. Mag.*, vol. 47, no. 12, pp. 42–49, 2009.

[6] *Cisco report, Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2012-2017*, 2011.

[7] C.-H. Yu, O. Tirkkonen, K. Doppler, and C. Ribeiro, "Power optimization of device-to-device communication underlaying cellular communication," in *Proc. IEEE International Conference on Communications, 2009. (ICC 09)*, 2009.

[8] X. Chen, L. Chen, M. Zeng, X. Zhang, and D. Yang, "Downlink resource allocation for device-to-device communication underlaying cellular networks," in *Proc. IEEE 23rd International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC), 2012*, 2012, pp. 232–237.

[9] M. Duarte, A. Sabharwal, V. Aggarwal, R. Jana, K. K. Ramakrishnan, C. W. Rice, and N. K. Shankaranarayanan, "Design and characterization of a full-duplex multi-antenna system for WiFi networks," *IEEE Trans. Veh. Technol.*, 2012, available: http://arxiv.org/abs/1210.1639.

[10] M. Duarte and A. Sabharwal, "Full-duplex wireless communications using off-the-shelf radios: Feasibility and first results," in *Proc. 44th Asilomar Conference on Signals, Systems and Computers*, 2010, pp. 1558–1562.

[11] I. Gradshteyn and I. Ryzhik, *Table of Integrals, Series, and Products, Seventh Edition.* Academic Press, 2007.

[12] J. Zhang, R. W. Heath, M. Kountouris, and J. G. Andrews, "Mode switching for the multi-antenna broadcast channel based on delay and channel quantization," *EURASIP J. Adv. Signal Process*, vol. 2009, pp. 1:1–1:15, Feb. 2009.

[13] N. S. Ferdinand, N. Rajatheva, and M. Latva-aho, "Performance analysis of two-way relay system with antenna correlation," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*, Houston, Tx, USA, 2011.

[14] J. Lee, H. Wang, J. G. Andrews, and D. Hong, "Outage probability of cognitive relay networks with interference constraints," *IEEE Trans. Wireless Commun.*, vol. 10, no. 2, pp. 390–395, 2011.