WAVEFORM OPTIMIZATION FOR OVERSAMPLED TRANSMULTIPLEXERS IN THE PRESENCE OF TIME-OFFSET

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ABSTRACT

In this paper we consider the case of an oversampled exponentially modulated transmultiplexer. We particularly focus on the interference created by a time offset impairment. We, firstly, simplify the expression of the Signal to Interference Ratio (SIR). Then, using this SIR expression as the optimization criterion, we provide perfect (PR) and nearly perfect (NPR) solutions that all outperform conventional OFDM.

***Index Terms*—** FMT; OFDM; Timing Offset; Transmultiplexer.

1. INTRODUCTION

Each new mobile communication standard has brought a new modulation system with respect to the previous one. Indeed if the 2G has retained the Gaussian Minimum Shift Keying (GMSK) scheme, 3G employs wideband Coded Division Multiplex Access (W-CDMA) while with the 4G the modulation depends on the context which is either DownLink (DL) or Uplink (UL). These constant evolutions are due to modifications of the available frequency bands and also to targeted services which are going more and more from voice to data. In a 5G perspective [1], [2], the current trend is to readopt a multicarrier modulation (MCM), as already the case in the 4G with the OFDM for the DL 3GPP LTE [3]. However, in spite of its important advantages, fast implementation algorithms and robustness in front of fading channels, OFDM leaves room to some other MCMs that can maintain these advantages and get rid of its main drawback, namely its $\sin(x)/x$ spectrum. Indeed this poor frequency localization makes it sensitive to any frequency impairment. More generally speaking, from a 5G point of view, it appears that a significant number of new scenarios, e.g. those related to Ultra Dense Networks [1], correspond to highly desynchronized time and frequency environments. It is therefore important to analyze the behavior of any proposed modulation scheme in relation with its robustness to time and frequency offsets.

In this paper, starting from a transmultiplexer (TMUX) formalism, we consider the case of a MCM which is either known as oversampled OFDM [4] or Filtered MultiTone (FMT) [5]. Its basic advantage, w.r.t OFDM, lies in its better spectrum behavior provided by its prototype filter that, due to the oversampling, does not need to rectangular. As in [6, 7, 8], our theoretical analysis focuses on the case of timing offset (TO) and carrier frequency offset (CFO). As illustrated in [6, 8], it is well-known that for Perfect Reconstruction (PR) systems, if there is no time and frequency offset, the interference is null while this is no longer the case when using, as in [7], a non PR prototype filter. For non-zero offsets an interference always occurs which is measured by the Signal to Interference Ratio (SIR).

Our first contribution is to provide a simplified expression of the SIR which is valid for PR and non PR (NPR) TMUX. Afterwards, we only focus on TO considering two cases. In a first one, our investigation is restricted to PR prototype filters having a length limited to the expansion/decimation factor of the TMUX, i.e. no overlapping between consecutive MCM symbols [9]. Then, differently from [6, 8], we go beyond the analysis step also providing a SIR optimization. In the second case, we consider longer prototype filters derived from the Squared Root Raised Cosine (SRRC) function. The SRRC is often the reference in the field but, when implemented with a finite length prototype filter, it looses its perfect orthogonality property being only NPR. In this paper we extend the analysis provided in [7] for the SRRC. Indeed, we introduce a procedure that, for given filter length and TO, selects the optimal SRRC roll-off. These optimal non PR solutions are compared with the short PR solutions. Our paper is organized as follows. Section 2 gives a general presentation of our system and channel model. Our simplified SIR expression is derived in Section 3. In Section 4, we present our analysis and optimization steps for the short PR and the longer, nearly PR, SRRC prototype filters. Our optimization results are reported in Section 5. Section 6 gives our conclusions.

THIS WORK HAS BEEN PERFORMED IN THE FRAMEWORK OF THE FP7 PROJECT ICT-317669 METIS.

2. CHANNEL AND SYSTEM MODEL

Let us denote by M and N two integers such that $M \leq N$. Our channel/system model is depicted in Fig. 1. The transmission system corresponds to a TMUX in which the expansion and decimation factors are equal to N. $F_m(z)$ and $H_m(z)$, $0 \leq m \leq M-1$ denote the filters of the synthesis (SFB) and analysis filter bank (AFB), respectively. In a general setting, they correspond to the z-transform of their complex impulse response

$$F_m(z) = \sum_{k\in\mathbb{Z}} f_m[k]z^{-k},\ H_m(z) = \sum_{k\in\mathbb{Z}} h_m[k]z^{-k}. \quad (1)$$

Denoting by $x_m[n], n \in \mathbb{Z}, 0 \leq m \leq M-1$, the input signals of the SFB and by $s[n],\ n \in \mathbb{Z}$, the resulting output signal, we get

$$s[n] = \sum_{m=0}^{M-1} \sum_{k\in\mathbb{Z}} x_m[k]\ f_m[n-kN]. \quad (2)$$

In order to mimic two common realization impairments, we suppose that the transmission channel introduces a time (TO) and a carrier frequency offset (CFO). Then, its output signal $\tilde{s}[n], n \in \mathbb{Z}$ is given by

$$\tilde{s}[n] = e^{2j\pi\mu n}\ s[n-d],\ n \in \mathbb{Z}, \quad (3)$$

where $j^2 = -1$ and $d \geq 0$ denotes the discrete TO while μ is a real-valued number representing the frequency offset. We will assume that μ can be expressed as $\mu = \dfrac{\varepsilon}{M}$ where $\varepsilon \in \mathbb{Z}$ denotes the *normalized CFO*. When $\varepsilon = 0$ and $d = 0$, the channel is said to be distortion-free and then $\tilde{s}[n] = s[n],\ n \in \mathbb{Z}$. Indeed, as here we focus on the TO and CFO impact, we also assume we have a noise-free transmission.

At the receiver side, the output signals of the AFB are given by

$$\hat{x}_m[n] = \sum_{l\in\mathbb{Z}} \tilde{s}[l]\ h_m[nN-l]. \quad (4)$$

In the following, we only focus on *orthogonal* TMUX, i.e. the AFB corresponds to a set of filters that are matched to the ones of the SFB. Otherwise said the coefficients of the $F_m(z)$ and $H_m(z)$ filters are linked together by the following equation $h_m[n] = \overline{f_m[-n]},\ 0 \leq m \leq M-1,\ n \in \mathbb{Z}$.

Furthermore, the TMUX uses an *exponential modulation*, meaning that the $F_m(z)$ filters are given by $F_m(z) = P(\omega_M^m z),\ \ 0 \leq m \leq M-1$, where ω_M denotes the Mth root of unity $\omega_M = e^{\frac{-2j\pi}{M}}$ and $P(z) = \sum_{n\in\mathbb{Z}} p[n]z^{-n}$ is a filter with real-valued coefficients, $p[n], n \in \mathbb{Z}$. For sake of simplicity, we use the notation ω instead of ω_M expressing the impulse response of the SFB and AFB by

$$f_m[n] = \omega^{-nm}p[n], h_m[n] = \omega^{-nm}p[-n]. \quad (5)$$

Then, the relation (3) can also be rewritten as

$$\tilde{s}[n] = \omega^{-\varepsilon n}\ s[n-d],\ n \in \mathbb{Z} \quad (6)$$

Combining (2), (6), (4) and (5), and after some index substitutions in the summations, we get

$$\hat{x}_m[n] = \omega^{-\varepsilon nN} \sum_{m'=0}^{M-1} \sum_{k\in\mathbb{Z}} \omega^{-m'[(n-k)N-d]} \quad (7)$$
$$\times \left(\sum_{l\in\mathbb{Z}} p[l]\ p[l - \frac{\varepsilon + m' - m}{M}]\ \omega^{-(\varepsilon+m'-m)l} \right) x_{m'}[k].$$

Then, defining the ambiguity function of p by

$$A_P[x,y] = \sum_{l\in\mathbb{Z}} p[l]\ p[l+x]\ e^{2j\pi ly}, \quad (8)$$

(7) is reformulated as follows

$$\begin{aligned} \hat{x}_m[n] &= \omega^{-\varepsilon nN} \sum_{m'=0}^{M-1} \sum_{k\in\mathbb{Z}} \omega^{-m'[(n-k)N-d]} \quad (9) \\ &\times\ A_P\left[(n-k)N-d, -\frac{\varepsilon+m'-m}{M}\right]\ x_{m'}[k]. \end{aligned}$$

The relation (9) can be rewritten as the summation of two components

$$\hat{x}_m[n] = a(m,n)x_m[n] + J(m,n), \quad (10)$$

with

$$\begin{aligned} a(m,n) &= \omega^{-\varepsilon nN+md} A_P\left[-d, -\frac{\varepsilon}{M}\right], \quad (11) \\ J(m,n) &= \omega^{-\varepsilon nN} \sum_{(k,m')\neq(0,0)} \omega^{-m'[(n-k)N-d]} \\ &\times\ A_P\left[(-kN-d, -\frac{\varepsilon+m'}{M}\right]\ x_{m'}[k], \quad (12) \end{aligned}$$

where (11) is the channel/system multiplicative factor while (12) contains all the interference terms.

3. SIGNAL TO INTERFERENCE RATIO

Assuming, as usual for symbol constellations, the input signals correspond to random variables being independent and identically distributed (iid), the Signal to Interference Ratio (SIR) can be expressed as a function of d and ε given by

$$\mathrm{SIR}(d,\varepsilon) = \frac{E\{|a(m,n)|^2\}}{E\{|J(m,n)|^2\}}, \quad (13)$$

where $E\{.\}$ denotes expectation. The $x_m[k]$ being iid, we get

$$\mathrm{SIR}(d,\varepsilon) = \frac{\left|A_P[-d, -\frac{\varepsilon}{M}]\right|^2}{\sum_{(k,m')\neq(n,m)} \left|A_P\left[(n-k)N-d, -\frac{\varepsilon+m'-m}{M}\right]\right|^2}.$$

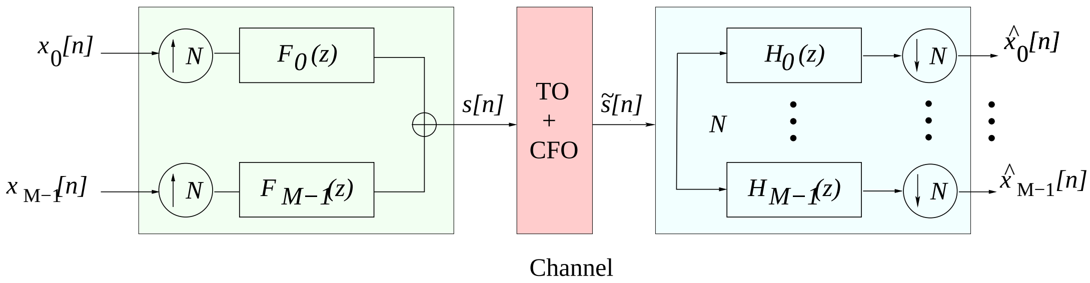

Fig. 1: Transmultiplexer with M and N parameters.

Using the fact that the $A_P(x,y)$ function is periodic in y with period 1 and also that ε is assumed to be integer, we can write

$$\text{SIR}(d,\varepsilon) = \frac{\left|A_P[-d,-\frac{\varepsilon}{M}]\right|^2}{S(d) - \left|A_P[-d,-\frac{\varepsilon}{M}]\right|^2}, \quad (14)$$

where $S(d)$ is independent from ε and given by

$$S(d) = \sum_{n\in\mathbb{Z}} \sum_{m=0}^{M-1} \left| A_P \left[nN - d, \frac{m}{M} \right] \right|^2. \quad (15)$$

Here we recover a similar expression to the ones given in [6, (22)] and [8].

In order to speed up the computation of (15), we now propose to reformulate the expression of $S(d)$ in such a way that the use of complex numbers is no longer required.

Setting $l = kM + s$ with $0 \le s \le M-1$, we get

$$A_P \left[kN - d, \frac{m}{M} \right] = \sum_{s=0}^{M-1} E_{s,n,d}\, \omega^{-sm}. \quad (16)$$

where $E_{s,n,d}$ is defined by

$$E_{s,n,d} = \sum_{k\in\mathbb{Z}} p[s+kM]\; p[s+kM+nN-d]. \quad (17)$$

The relations (16) for $0 \le s \le M-1$ can be written in a matrix form

$$\boldsymbol{F}_M\, \boldsymbol{E}_{n,d} = \boldsymbol{V}_{n,d}, \quad (18)$$

where $\boldsymbol{E}_{n,d}$ is the M-vector whose entries are $E_{s,n,d},\ 0 \le s \le M-1$, $\boldsymbol{V}_{n,d}$ is the M-vector whose entries are $A_P\left[kN-d,\frac{m}{M}\right],\ 0 \le m \le M-1$ and $\boldsymbol{F}_M$ is the discrete Fourier matrix of size M. According to Plancherel's theorem, we then get

$$\sum_{m=0}^{M-1} \left| A_P \left[nN - d, \frac{m}{M} \right] \right|^2 = M \sum_{s=0}^{M-1} E_{s,n,d}^2, \quad (19)$$

since the $p[n],\ n \in \mathbb{Z}$ coefficients are real numbers. Consequently

$$S(d) = M \sum_{n\in\mathbb{Z}} \sum_{s=0}^{M-1} E_{s,n,d}^2. \quad (20)$$

The TMUX is said to have the *perfect reconstruction* property (PR) if, for a distortion-free channel, there exists a constant $\alpha \neq 0$ such that for each input signal we get $\hat{x}_m[n] = \alpha x_m[n],\ n \in \mathbb{Z},\ 0 \le m \le M-1$. According to (11) and (12), this condition is therefore equivalent to the following conditions

$$A_P \left[nN, \frac{m}{M} \right] = \alpha \delta_{n,0}\ \delta_{m,0},\ n \in \mathbb{Z},\ 0 \le m \le M-1, \quad (21)$$

where $\delta_{p,q}$ denotes the Kronecker symbol. In a vector form this amounts to consider 2 cases for the vector $\boldsymbol{V}$: $\boldsymbol{V}_{0,0} = \alpha[1,0,0,\ldots,0]^T$, where $[.]^T$ denotes transposition, if $n=0$ and $\boldsymbol{V}_{n,0} = [0,0,0,\ldots,0]^T$ if $n \neq 0$. According to the equality (18), we get $\boldsymbol{E}_{0,0} = \frac{\alpha}{M}\ [1,1,\ldots,1]^T$ and $\boldsymbol{E}_{n,0} = [0,0,\ldots,0]^T$. Then for $n \in \mathbb{Z},\ 0 \le s \le M-1$, we get

$$E_{s,n,0} = \sum_{k\in\mathbb{Z}} p[s+kM]\; p[s+kM+nN] = \frac{\alpha}{M}\delta_{n,0}. \quad (22)$$

This corresponds to the PR relations derived at first in [10, (64)] for short-time Fourier analysis and reused in [11, (10)] for PR oversampled OFDM systems.

4. THE TWO FAMILIES OF PROTOTYPE FILTERS

In this section we focus on the case of a TO impairment, therefore, based on (14), the problem is to find the prototype filter maximizing $\text{SIR}(d,0)$ for given values of d, with naturally $d > 0$ in the PR case. In addition to these two families we also characterize the OFDM behavior.

4.1. Short PR prototype filters

As in [9], for a given integer Δ the oversampling ratio is such that $\frac{N}{M} = \frac{N_0}{M_0}$ with $N = \Delta N_0$ and $M = \Delta M_0$. To get a maximum spectral efficiency we also set $N_0 = M_0 + 1$, then a *short* PR prototype filter is a filter of length $L = \Delta L_0 = N$ satisfying (22) and then the unique solution can be expressed using a set of Δ angles [12, (78)]

$$p[i] = \cos\theta_i,\ \ p[M+i] = \sin\theta_i,\ 0 \le i \le \Delta - 1,$$
$$p[k] = 1,\ \Delta \le k \le M-1. \quad (23)$$

As a matter of example we can illustrate the case with a linear phase prototype filter, that we call the *podium* filter defined as follows

$$p[n] = p[L-1-n] = \begin{cases} 1, & \Delta \leq n \leq M_0\Delta - 1, \\ \frac{\sqrt{2}}{2}, & 0 \leq n \leq \Delta - 1. \end{cases} \quad (24)$$

For this simple example we can get a closed-form expression of the SIR which, for $M_0 \geq 2$ and $1 \leq d \leq \Delta$, reads as

$$\text{SIR}_{P(M_0,\Delta)}(d,0) = \frac{\left(M - (2-\sqrt{2})d\right)^2}{d\left((\frac{13}{4} - 2\sqrt{2})M - (6 - 4\sqrt{2})d\right)} \quad (25)$$

while if $M_0 = 1$, $1 \leq d \leq \Delta$, it becomes

$$\text{SIR}_{P(1,\Delta)}(d,0) = \frac{2\Delta - d}{d}. \quad (26)$$

Beyond this toy example, we can use the compact representation (CR) [9], then, instead of dealing with the Δ angles (23), the optimization for various design criteria can be carried out over the set of d_c parameters of the CR [9, (6)]. The great advantage is that even with $d_c \ll \Delta$ we can get nearly optimal solutions. In the present case the objective function to maximize is SIR$(1,0)$ and it leads to the *best prototype filters*.

Observing the behavior of the optimal set of coefficients, we noticed the angles are very close to a trapezium distribution. So, for $M_0 \geq 1$ and $\Delta \geq 1$, we now define another short PR and symmetrical prototype filter, named *trapezium* filter, as follows

$$p[n] = p[L-1-n] = \begin{cases} \cos\left(\frac{\pi}{2}[1 - \frac{2n+1}{2\Delta}]\right), & 0 \leq n \leq \Delta - 1, \\ 1, & \Delta \leq n \leq M_0\Delta - 1. \end{cases}$$

4.2. The Squared Root Raised Cosine Filter

The squared root raised cosine function with roll-off parameter r, $0 < r \leq 1$, is given either by its Fourier transform

$$F(f) = \begin{cases} 1, & |f| \leq \frac{1-r}{2}, \\ \cos\left(\frac{\pi}{2r}(|f| - \frac{1-r}{2})\right), & \frac{1-r}{2} \leq |f| \leq \frac{1+r}{2}, \\ 0, & |f| \geq \frac{1+r}{2}, \end{cases} \quad (27)$$

or, equivalently, by its time representation

$$f(t) = \frac{\sin(\pi(1-r)t) + 4rt\cos(\pi(1+r)t)}{\pi t(1 - 16r^2t^2)}. \quad (28)$$

For a given parameter $N > 0$ and time sampling $T_s = 1/N$, the discrete-time version of the SRRC prototype filter with even length, $L > N$, writes as

$$p[L/2+i] = p[L/2-i-1] = f\left(\frac{2i+1}{2N}\right), \; 0 \leq i < \frac{L}{2}. \quad (29)$$

When equipped with this SRRC prototype filter, the PR property can only be attained if N and L tend to infinity and for a choice of a roll-off factor such that $r_\infty = (N-M)/M$ [7]. In a practical setting, L is of a finite value and SIR$(0,0)$ is bounded. Nevertheless, using a fast inspection method we can find the optimal roll-off value, r_{opt}, that, as for PR prototypes, maximizes SIR$(1,0)$.

In Fig. 2, we can see that for lengths being equal to 4 to 7 times N, there is a significant SIR gain compared to what we get when $r = r_\infty$.

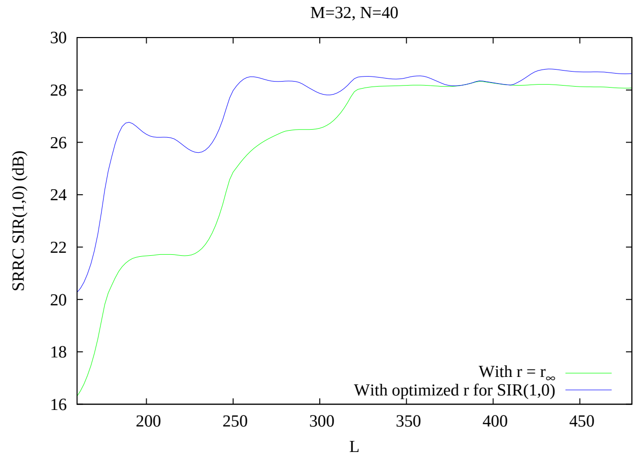

Fig. 2: Optimization of r for the SRRC filter with $M = 32$, $N = 40$, $160 \leq L \leq 480$ and cost function SIR$(1,0)$.

5. OPTIMIZATION RESULTS

For the three tested oversampling ratio, we can see in Fig. 3 to 5 that the trapezium prototype filter reaches the optimal bound attainable for short PR filters. Furthermore, if the delay is not too large it also provides the globally best results. In Figs. 4, 5, we also see that for the SRRCs the roll-off optimization may provide an interesting gain. Note also that beyond a certain delay, 6 to 10%, they give better results than the short PR prototype filters that have been optimized for 2.5 and 0.17%, respectively. OFDM, and its rectangular filter with length $L = M$, is also included in this comparison using the SIR expression provided in [13]

$$\text{SIR}(d,0) = \frac{(M-d)^2}{d(2M-d)}, \; 1 \leq d \leq M. \quad (30)$$

OFDM is most often the worst solution, even worst than the podium filter. However, in a practical setting, any interference could be avoided, adding a cyclic prefix, i.e. with CP-OFDM. This indeed results in an horizontal translation of the OFDM SIR curve [7], but in the meantime to a significant reduction of the spectral efficiency.

6. CONCLUSION

The optimization of a transmultiplexer in the presence of multiple channel impairments is a difficult task that will be investigated in our future studies. Nevertheless, in this paper, limiting the analysis to the time-offset impairment, we have shown that simple solutions can be obtained for PR short systems and for the NPR SRRC prototype filter as well.

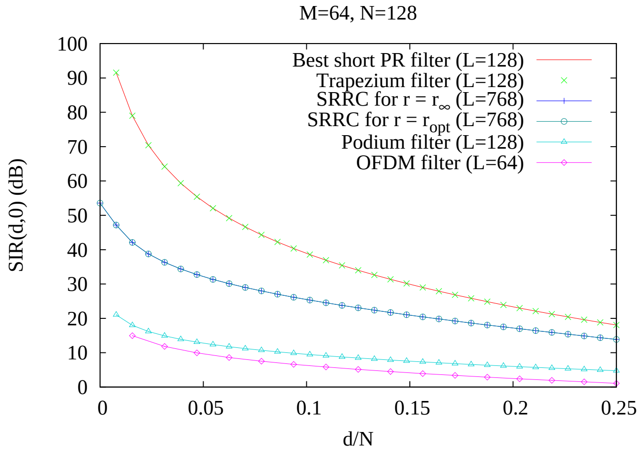

Fig. 3: SIR with $M = 64$, $N = 128$ and $0 \leq d/N \leq 0.25$ ($r_{opt} = r_{\infty} = 1$).

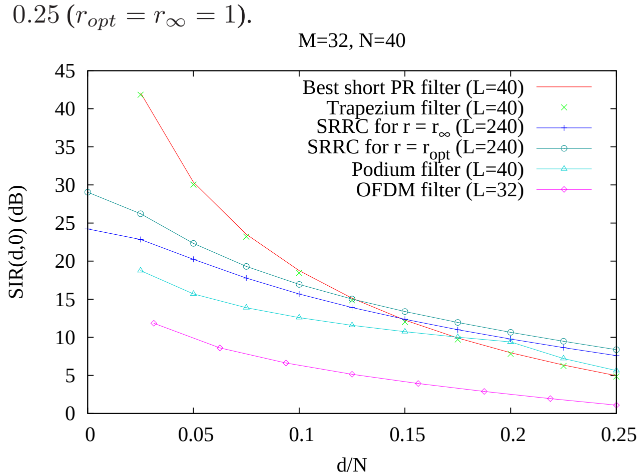

Fig. 4: SIR comparison for $M = 32, N = 40$ and $0 \leq d/N \leq 0.25$ ($r_{opt} = 0.351230$).

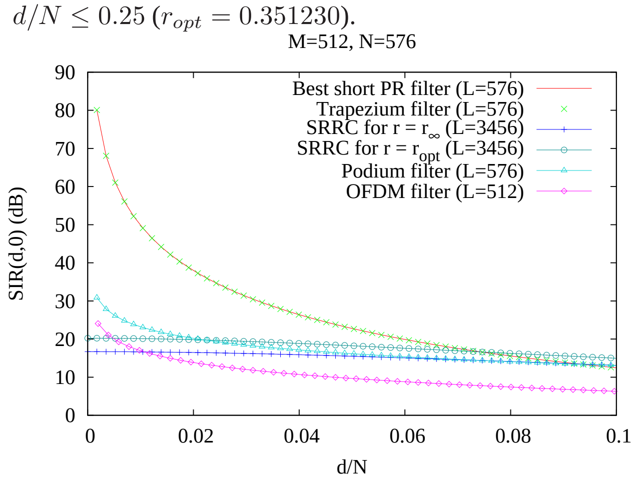

Fig. 5: SIR with $M = 512$, $N = 576$ and $0 \leq d/N \leq 0.1$ ($r_{opt} = 0.269053$) .

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