# MMSE Interference Estimation in LTE Networks


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***Abstract*—We present a statistical approach for estimating the interference coupling coefficients in an LTE network based on a set of various measurements available at the network and terminal level. The proposed approach combines the measurements with prior information (spatial correlation among interference links) and takes into account measurement uncertainty. The result is a simple closed-form estimator that allows for fast real-time interference estimation.**

***Index Terms*—Cellular networks, LTE, inter-cell interference, spatial correlation, MMSE, least squares.**


## I. Introduction

Long Term Evolution Advanced (LTE-A) heterogeneous networks (HetNets) offer improved coverage, capacity, and spectral efficiency compared with traditional LTE systems, thanks to a dense deployment of novel network elements such as pico/femto base stations (BS) and relay nodes. However, such dense and often unplanned deployment with full frequency reuse (reuse factor 1) may lead to critical inter-cell interference problems. Estimating the interference level of the network links becomes therefore a crucial task. In particular, knowledge of the long-term interference structure in form of the interference matrix may be beneficially exploited by the network to implement self-adaptation mechanisms, or to enable direct device-to-device (D2D) communication mode when necessary (see for instance the discussuion in [1] and references therein).

Several inter-cell interference management schemes have been proposed in LTE-A under the name of "enhanced inter-cell interference coordination (eICIC)" [2]–[4]. An interference management approach based on "cognitive BSs" is suggested in [5]. Other works more closely related to the task of estimation of the interference matrix are: [6], dealing with estimation of the spatial covariance matrix; [7], using a control theory-based approach; and [8], where available measurements at the network as well as user equipment (UE) level are incorporated with physical bounds as a set-theoretic estimation approach.

In this paper, we consider the same available measurements as in [8], but we propose a statistical estimation approach. By doing so, we are able to combine the measurements with (i) prior knowledge of spatial long-term correlation of the interference links (due to correlated slow fading channels), and (ii) statistical knowledge of measurement uncertainty. This stands in clear contrast to [8] where no statistical knowledge about the underlying interference process is exploited, making the scheme more suitable and robust to scenarios in which this knowledge is not available and cannot be reliably learned. The approach of [8] is therefore not directly comparable to that proposed in this paper since the latter presumes an additional prior knowledge, while the former is a distributed online learning scheme that continuously learn the interference structure as new measurements arrive.

Given some physical-layer measurements available in the LTE downlink and uplink, in Section III of this paper, we derive a linear model corrupted by measurement uncertainty. The model relates the measurements to the so-called channel gain matrix, which in turn can be used to reconstruct the sought interference matrix. As shown in Section III, this model can be used to obtain optimal linear Minimum Mean Square Error (MMSE) estimator, provided that the prior distribution of the interference matrix as well as the uncertainty distribution are both Gaussian (in linear scale). Since slow fading caused by shadowing is often assumed to have a log-normal distribution, we adopt in Section IV a more realistic model in which the prior interference distribution is log-normal (typical model for shadowing) and the uncertainty distribution is Gaussian in dB scale (accounting for quantization errors and offsets in the reported values). Unfortunately, when written in dB scale, the model becomes non-linear. Therefore, we propose a closed-form "linearized" MMSE estimator that is based on the first order approximation of the non-linear model. The proposed approaches are evaluated in Section V, while complexity issues are discussed in Section VI. Conclusion is drawn in the last section.

## II. Problem Formulation

Given a cellular network with $K_\mathsf{U}$ users and $K_\mathsf{B}$ base stations, we use $\mathbf{A} \in \{0,1\}^{K_\mathsf{U} \times K_\mathsf{B}}$ and $\mathbf{H} \in \mathbb{R}^{K_\mathsf{U} \times K_\mathsf{B}}$ to denote the assignment matrix and the channel (power) gain matrix, respectively. While the assignment matrix describes the assignment of the users to the base stations, the channel gain matrix includes all power attenuation effects for each transmitter-receiver pair. Therefore, given $\mathbf{A}$ and some power


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allocation vector, the matrix $\mathbf{A}^T\mathbf{H}$ determines the interference power at all receivers. Since $\mathbf{A}$ is usually known in the network, the interference matrix can be easily computed whenever $\mathbf{H}$ is known so that our goal in this paper is to estimate $\mathbf{H}$ or, equivalently, $\mathbf{h} \triangleq \mathrm{vec}(\mathbf{H}) \in \mathbb{R}^K$ where $K \triangleq K_{\mathsf{B}}K_{\mathsf{U}}$. We assume a prior distribution of $\mathbf{h}$ is available, with $\overline{\mathbf{h}} \triangleq \mathbb{E}[\mathbf{h}]$ and $\mathbf{C}_{\mathbf{h}} \triangleq \mathbb{E}[(\mathbf{h}-\overline{\mathbf{h}})(\mathbf{h}-\overline{\mathbf{h}})^T]$. The prior mean $\overline{\mathbf{h}}$ may be given by the path loss (assuming known positions of UEs and BSs and known path loss exponent). A general model for the covariance $\mathbf{C}_{\mathbf{h}}$ may be expressed as follows. We first note that the relation between indexes of $\mathbf{H}$ and $\mathbf{h} = [h_1, \dots, h_K]$ can be written as

$$[\mathbf{H}]_{u,b} = h_{K_{\mathsf{U}}(b-1)+u}, \tag{1}$$

therefore the user $u$ and BS $b$ corresponding to the $i$-th position of vector $\mathbf{h}$ can be retrieved by two simple functions,

$$b(i) \triangleq \lceil i/K_{\mathsf{U}} \rceil, \tag{2}$$

$$u(i) \triangleq i - K_{\mathsf{U}}(b(i)-1). \tag{3}$$

Hence, any $(i,j)$-th element of the covariance matrix can be expressed as

$$[\mathbf{C}_{\mathbf{h}}]_{i,j} = \rho e^{-d(\{b(i),u(i)\},\{b(j),u(j)\})}, \tag{4}$$

where $\rho$ is a positive constant and $d(\cdot,\cdot)$ is a properly chosen distance function between two BS-UE communication links. It may be assumed that only nodes served by the same BS are correlated, in which case $d(\{b,u\},\{b',u'\}) = \infty$ if $b \neq b'$. For a more concrete example of the correlation model, see Section V.

A complete list of the physical-layer measurements available in the LTE uplink and downlink is reported in [9]. Among these, particularly relevant for interference estimation are the *reference signal received power (RSRP)* [9, Section 5.1.1], measured by the UE in the downlink, and the *uplink interference (ULI) power* [9, Section 5.2.2]. In the following, it is assumed that the thermal noise power (also reported in the uplink, see [9, Section 5.2.3]) is subtracted from ULI measurements. Similarly, UEs are able to measure the *downlink interference (DLI) power*, by subtracting the RSRP and the known thermal noise power from their RSSI measurements [9, Section 5.1.3].

It is then assumed that the channel powers are *reciprocal*, i.e., $\mathbf{H}$ is the same in the UL and in the DL[1]. Without measurement uncertainty and in linear scale, the measurements can be written as follows.

- RSRP measurements:

$$r_i = p_i h_i, \quad i \in \{1, \dots, K\}, \tag{5}$$

  where $p_i$ is the reference signal transmit power of BS $b(i)$.

- ULI measurements:

$$\phi_b^{\mathsf{UL}} = \sum_{u=1}^{K_{\mathsf{U}}} p_u^{\mathsf{UL}} [\mathbf{H}]_{u,b}(1-[\mathbf{A}]_{u,b}), \quad b \in \{1, \dots, K_{\mathsf{B}}\}, \tag{6}$$

  where $p_u^{\mathsf{UL}}$ is the uplink transmit power of user $u$ (assumed as known at BS $b$).

- DLI measurements:

$$\phi_u^{\mathsf{DL}} = \sum_{b=1}^{K_{\mathsf{B}}} p_b^{\mathsf{DL}} [\mathbf{H}]_{u,b}(1-[\mathbf{A}]_{u,b}), \quad u \in \{1, \dots, K_{\mathsf{U}}\}, \tag{7}$$

  where $p_b^{\mathsf{DL}}$ is the downlink transmit power of BS $b$ (assumed as known at user $u$).

All these measurements, in practice, are affected by statistical uncertainties, as discussed in the following sections.

## III. MMSE ESTIMATOR WITH LINEAR MODEL

We first introduce a Gaussian linear model for the measurement uncertainties and for the prior probability density function (p.d.f.) of $\mathbf{h}$. This model simplifies the mathematical analysis. Under this model, the vector of $K$ RSRP measurements can be written as

$$\mathbf{r} = \mathrm{Diag}(\mathbf{p})\mathbf{h} + \mathbf{n}^{\mathsf{RSRP}} \tag{8}$$

where $\mathbf{p} \triangleq [p_1, \dots, p_K]^T$ is the reference signal transmit power vector and $\mathbf{n}_{\mathsf{RSRP}} \in \mathbb{R}^K \sim \mathcal{N}(0, \sigma^2_{\mathsf{RSRP}}\mathbf{I})$ is a vector of Gaussian distributed, uncorrelated measurement uncertainties. We assume here that the channel entries have a joint Gaussian distribution in linear scale, such that $\mathbf{h} \sim \mathcal{N}(\overline{\mathbf{h}}, \mathbf{C}_{\mathbf{h}})$.

For ULI measurements, we rewrite (6) as

$$\phi_b^{\mathsf{UL}} = (\overline{\mathbf{a}}_b \odot \mathbf{p}^{\mathsf{UL}})^T \cdot [\mathbf{h}]_{K_{\mathsf{U}}(b-1)+1}^{bK_{\mathsf{U}}} \tag{9}$$

$$= \left[\mathbf{e}_b \otimes (\overline{\mathbf{a}}_b \odot \mathbf{p}^{\mathsf{UL}})\right]^T \cdot \mathbf{h}\,, \tag{10}$$

where $\mathbf{p}^{\mathsf{UL}} \triangleq [p_1^{\mathsf{UL}}, \dots, p_{K_{\mathsf{U}}}^{\mathsf{UL}}]^T$, $\overline{\mathbf{a}}_b$ is the $b$-th column of $\overline{\mathbf{A}}$, i.e. the 1-complement of the assignment matrix $\mathbf{A}$, $\odot$ denotes element-wise product, $\otimes$ denotes Kronecker product, $[\mathbf{v}]_i^j$ denotes a sub-vector of $\mathbf{v}$ from index $i$ through $j$, and $\mathbf{e}_b \in \{0,1\}^{K_{\mathsf{B}}}$ is a vector of all zero elements except a one in position $b$. From (10), and introducing an uncertainty vector $\mathbf{n}^{\mathsf{ULI}} \in \mathbb{R}^{K_{\mathsf{B}}} \sim \mathcal{N}(\mathbf{0}, \sigma^2_{\mathsf{ULI}}\mathbf{I})$, we can write the vector of $K_{\mathsf{B}}$ ULI measurements as

$$\boldsymbol{\phi}^{\mathsf{UL}} = \left[\mathbf{e}_1 \otimes (\overline{\mathbf{a}}_1 \odot \mathbf{p}^{\mathsf{UL}}), \dots, \mathbf{e}_{K_{\mathsf{B}}} \otimes (\overline{\mathbf{a}}_{K_{\mathsf{B}}} \odot \mathbf{p}^{\mathsf{UL}})\right]^T \cdot \mathbf{h} + \mathbf{n}^{\mathsf{ULI}} \tag{11}$$

$$= \left[\mathbf{I}_{K_{\mathsf{B}}} * \left(\mathrm{Diag}(\mathbf{p}^{\mathsf{UL}}) \cdot \overline{\mathbf{A}}\right)\right]^T \cdot \mathbf{h} + \mathbf{n}^{\mathsf{ULI}}\,, \tag{12}$$

where $\mathbf{I}_{K_{\mathsf{B}}}$ is the identity matrix and $*$ denotes Khatri-Rao product[2] [10].

Similarly, for the downlink, we obtain

$$\boldsymbol{\phi}^{\mathsf{DL}} = \left[\left(\mathrm{Diag}(\mathbf{p}^{\mathsf{DL}}) \cdot \overline{\mathbf{A}}^T\right) * \mathbf{I}_{K_{\mathsf{U}}}\right]^T \cdot \mathbf{h} + \mathbf{n}^{\mathsf{DLI}}, \tag{13}$$

[1]Note the assumption of channel reciprocity only involves the power, but not the phase.

[2]The Khatri-Rao product is defined as the column-wise Kronecker product between two matrices: given $\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_n] \in \mathbb{R}^{m\times n}$ and $\mathbf{Y} = [\mathbf{y}_1, \dots, \mathbf{y}_n] \in \mathbb{R}^{p\times n}$, then $\mathbf{X} * \mathbf{Y} \triangleq [\mathbf{x}_1 \otimes \mathbf{y}_1, \dots, \mathbf{x}_n \otimes \mathbf{y}_n] \in \mathbb{R}^{mp\times n}$

where $\mathbf{p}^{\mathsf{DL}} \triangleq [p_1^{\mathsf{DL}}, \ldots, p_{K_{\mathsf{B}}}^{\mathsf{DL}}]^T$ and $\mathbf{n}^{\mathsf{DLI}} \in \mathbb{R}^{K_{\mathsf{U}}} \sim \mathcal{N}(\mathbf{0}, \sigma_{\mathsf{DLI}}^2 \mathbf{I})$.

By using the above results, all the available observations can be combined into a simple linear model,

$$\underbrace{\begin{bmatrix} \mathbf{r} \\ \boldsymbol{\phi}^{\mathsf{UL}} \\ \boldsymbol{\phi}^{\mathsf{DL}} \end{bmatrix}}_{\triangleq \mathbf{y}} = \underbrace{\begin{bmatrix} \mathrm{Diag}(\mathbf{p}) \\ \left[\mathbf{I}_{K_{\mathsf{B}}} * \left(\mathrm{Diag}(\mathbf{p}^{\mathsf{UL}}) \cdot \overline{\mathbf{A}}\right)\right]^T \\ \left[\left(\mathrm{Diag}(\mathbf{p}^{\mathsf{DL}}) \cdot \overline{\mathbf{A}}^T\right) * \mathbf{I}_{K_{\mathsf{U}}}\right]^T \end{bmatrix}}_{\triangleq \mathbf{W}} \mathbf{h} + \underbrace{\begin{bmatrix} \mathbf{n}^{\mathsf{RSRP}} \\ \mathbf{n}^{\mathsf{ULI}} \\ \mathbf{n}^{\mathsf{DLI}} \end{bmatrix}}_{\triangleq \mathbf{n}}, \tag{14}$$

with $\mathbf{y}$, $\mathbf{n} \in \mathbb{R}^{K+K_{\mathsf{U}}+K_{\mathsf{B}}}$ and $\mathbf{W} \in \mathbb{R}^{(K+K_{\mathsf{U}}+K_{\mathsf{B}})\times K}$. Note that $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \mathbf{D})$ with

$$\mathbf{D} \triangleq \mathrm{Diag}\left(\left[\sigma_{\mathsf{RSRP}}^2 \mathbf{1}_K^T, \sigma_{\mathsf{ULI}}^2 \mathbf{1}_{K_{\mathsf{B}}}^T, \sigma_{\mathsf{DLI}}^2 \mathbf{1}_{K_{\mathsf{U}}}^T\right]^T\right). \tag{15}$$

At this point we can derive the MMSE estimator. Thanks to the Gaussian distribution of $\mathbf{h}$ and $\mathbf{n}$, we have the following property: *The optimal MMSE estimator is equivalent to the linear MMSE (LMMSE)*. Thus,

$$\hat{\mathbf{h}}^{\mathsf{MMSE}} = \hat{\mathbf{h}}^{\mathsf{LMMSE}} = \overline{\mathbf{h}} + \mathbf{C}_{\mathbf{h}} \mathbf{W}^T (\mathbf{W}\mathbf{C}_{\mathbf{h}}\mathbf{W}^T + \mathbf{D})^{-1} (\mathbf{y} - \mathbf{W}\overline{\mathbf{h}}). \tag{16}$$

This estimator exploits the prior knowledge of the statistics of $\mathbf{h}$ and of $\mathbf{n}$. An alternative estimator is the simple least squares (LS) estimator, which is given by

$$\hat{\mathbf{h}}^{\mathsf{LS}} = (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}^T\mathbf{y} \tag{17}$$

and can be viewed as a maximum likelihood (ML) estimator without any prior information on $\mathbf{h}$ and $\mathbf{n}$.

## IV. MMSE ESTIMATOR WITH LOGARITHMIC MODEL

A more realistic model is given by assuming that all the variables are in dB scale, so that $\mathbf{h}$ has a log-normal joint distribution (which reflects log-normal shadow fading) and the measurement uncertainty is also Gaussian in dB scale (which accounts for rounding or quantization errors, bit errors in the reported data, scale offsets, etc.). In the following, variables in dB are distinguished from the corresponding variables in linear scale by an underline sign: e.g., $\underline{h}_i \triangleq 10\log(h_i)$, $[\underline{\mathbf{H}}]_{u,b} \triangleq 10\log([\mathbf{H}]_{u,b})$, etc. The symbol $\log$ denotes base-10 logarithm.

Under this new model, we assume the prior distribution of the interference vector to be Gaussian in dB scale: $\underline{\mathbf{h}} \sim \mathcal{N}(\overline{\underline{\mathbf{h}}}, \mathbf{C}_{\underline{\mathbf{h}}})$. The log-covariance $\mathbf{C}_{\underline{\mathbf{h}}} \triangleq \mathbb{E}[(\underline{\mathbf{h}} - \overline{\underline{\mathbf{h}}})(\underline{\mathbf{h}} - \overline{\underline{\mathbf{h}}})^T]$, representing spatial correlation, can be expressed by the same distance-based model (4) used in the linear case (with the difference that it now applies to values of $\underline{\mathbf{h}}$ in dB).

The vector of noisy RSRP measurements becomes

$$\underline{\mathbf{r}} = \underline{\mathbf{p}} + \underline{\mathbf{h}} + \underline{\mathbf{n}}^{\mathsf{RSRP}}, \tag{18}$$

where $\underline{\mathbf{n}}_{\mathsf{RSRP}} \sim \mathcal{N}(0, \sigma_{\mathsf{RSRP}}^2 \mathbf{I})$ is the measurement uncertainty, now Gaussian distributed in dB scale. While RSRP measurements are still linear under the logarithmic model, ULI and DLI measurements become non-linear. Therefore, the LMMSE estimator is no longer equivalent to the MMSE estimator, and the MMSE itself is difficult to compute explicitly. For example, in the uplink case (6), we have for $b \in \{1, \ldots, K_{\mathsf{B}}\}$,

$$\underline{\phi}_b^{\mathsf{UL}} = 10\log\left(\sum_{u=1}^{K_{\mathsf{U}}} (1 - [\mathbf{A}]_{u,b}) 10^{0.1(\underline{p}_u^{\mathsf{UL}} + [\underline{\mathbf{H}}]_{u,b})}\right). \tag{19}$$

In order to tackle this problem, we propose the following approach: we linearize the measurement model by a first-order Taylor expansion centered in the prior mean $\overline{\underline{\mathbf{h}}}$; in this way, we obtain a linear combination of the Gaussian variables $\underline{\mathbf{h}}$ and $\underline{\mathbf{n}}$, and we can perform LMMSE estimation of $\underline{\mathbf{h}}$. We name the proposed approach *linearized log-MMSE (LLMMSE)* estimator. Note that no linearization is necessary for RSRP measurements (18), to which LMMSE estimation can be applied directly.

The linearized ULI measurement vector is

$$\underline{\boldsymbol{\phi}}^{\mathsf{UL}} \approx \underline{\boldsymbol{\phi}}^{\mathsf{UL}}(\overline{\underline{\mathbf{h}}}) + \mathbf{J}_{\underline{\boldsymbol{\phi}}^{\mathsf{UL}}}(\overline{\underline{\mathbf{h}}}) \cdot (\underline{\mathbf{h}} - \overline{\underline{\mathbf{h}}}) + \underline{\mathbf{n}}^{\mathsf{ULI}}, \tag{20}$$

where $\underline{\boldsymbol{\phi}}^{\mathsf{UL}}(\overline{\underline{\mathbf{h}}})$ is the vector of logarithmic ULI measurements $\underline{\boldsymbol{\phi}}^{\mathsf{UL}} = [\underline{\phi}_1^{\mathsf{UL}}, \ldots, \underline{\phi}_{K_{\mathsf{B}}}^{\mathsf{UL}}]^T$, given element-wise by (19), computed in $\underline{\mathbf{h}} = \overline{\underline{\mathbf{h}}}$; $\mathbf{J}_{\underline{\boldsymbol{\phi}}^{\mathsf{UL}}}(\overline{\underline{\mathbf{h}}}) \in \mathbb{R}^{K_{\mathsf{B}}\times K}$ is the matrix of partial derivatives (Jacobian matrix) of $\underline{\boldsymbol{\phi}}^{\mathsf{UL}}$ with respect to $\underline{\mathbf{h}}$, computed in $\underline{\mathbf{h}} = \overline{\underline{\mathbf{h}}}$; and $\underline{\mathbf{n}}_{\mathsf{ULI}} \sim \mathcal{N}(0, \sigma_{\mathsf{ULI}}^2 \mathbf{I})$ is the measurement uncertainty, normally distributed in dB scale. The symbol $\approx$ represents a first-order approximation[3].

We next derive a closed form of the Jacobian matrix. By definition,

$$\mathbf{J}_{\underline{\boldsymbol{\phi}}^{\mathsf{UL}}}(\underline{\mathbf{h}}) \triangleq \begin{bmatrix} \frac{\partial \underline{\phi}_1^{\mathsf{UL}}}{\partial \underline{h}_1} & \cdots & \frac{\partial \underline{\phi}_1^{\mathsf{UL}}}{\partial \underline{h}_K} \\ \vdots & \ddots & \vdots \\ \frac{\partial \underline{\phi}_{K_{\mathsf{B}}}^{\mathsf{UL}}}{\partial \underline{h}_1} & \cdots & \frac{\partial \underline{\phi}_{K_{\mathsf{B}}}^{\mathsf{UL}}}{\partial \underline{h}_K} \end{bmatrix}. \tag{21}$$

As a consequence of the ULI measurement structure (19) and of the indexing logic (1), two properties immediately follow:

$$\frac{\partial \underline{\phi}_b^{\mathsf{UL}}}{\partial [\underline{\mathbf{H}}]_{u,b'}} = \frac{\partial \underline{\phi}_b^{\mathsf{UL}}}{\partial \underline{h}_{K_{\mathsf{U}}(b'-1)+u}} = 0 \quad \forall u, \forall b' \neq b, \tag{22}$$

$$\frac{\partial \underline{\phi}_b^{\mathsf{UL}}}{\partial [\underline{\mathbf{H}}]_{u,b}} = \frac{\partial \underline{\phi}_b^{\mathsf{UL}}}{\partial \underline{h}_{K_{\mathsf{U}}(b-1)+u}} = 0 \quad \forall u, b : [\mathbf{A}]_{u,b} = 1. \tag{23}$$

The other elements of the matrix are given by

$$\frac{\partial \underline{\phi}_b^{\mathsf{UL}}}{\partial [\underline{\mathbf{H}}]_{u,b}} = \frac{\partial \underline{\phi}_b^{\mathsf{UL}}}{\partial \underline{h}_{K_{\mathsf{U}}(b-1)+u}} \tag{24}$$

$$= \frac{(1 - [\mathbf{A}]_{u,b}) 10^{0.1(\underline{p}_u^{\mathsf{UL}} + [\underline{\mathbf{H}}]_{u,b})}}{\sum_{u'=1}^{K_{\mathsf{U}}} (1 - [\mathbf{A}]_{u',b}) 10^{0.1(\underline{p}_{u'}^{\mathsf{UL}} + [\underline{\mathbf{H}}]_{u',b})}} \tag{25}$$

$$= \frac{(1 - [\mathbf{A}]_{u,b}) p_u^{\mathsf{UL}} [\mathbf{H}]_{u,b}}{\sum_{u'=1}^{K_{\mathsf{U}}} (1 - [\mathbf{A}]_{u',b}) p_{u'}^{\mathsf{UL}} [\mathbf{H}]_{u',b}} \tag{26}$$

$$= \frac{(1 - [\mathbf{A}]_{u,b}) p_u^{\mathsf{UL}} [\mathbf{H}]_{u,b}}{\left[\mathbf{e}_b \otimes (\overline{\mathbf{a}}_b \odot \mathbf{p}^{\mathsf{UL}})\right]^T \cdot \mathbf{h}}. \tag{27}$$

[3]The error introduced by this approximation is $o(\underline{\mathbf{h}} - \overline{\underline{\mathbf{h}}})$. The leading error term can be quantified by a quadratic form involving the Hessian matrix.

Therefore, by algebraic manipulations similar to (10)-(12), we can express the Jacobian matrix as

$$\mathbf{J}_{\underline{\phi}^{\mathsf{UL}}}(\underline{\mathbf{h}}) = (\mathbf{Z}^{\mathsf{UL}})^{-1}\cdot\left[\mathbf{I}_{K_\mathsf{B}} * \left(\mathrm{Diag}(\mathbf{p}^{\mathsf{UL}})\cdot\overline{\mathbf{A}}\right)\right]^T\cdot\mathrm{Diag}(\mathbf{h}) \quad (28)$$

where

$$\mathbf{Z}^{\mathsf{UL}} \triangleq \mathrm{Diag}\left\{\left[\mathbf{I}_{K_\mathsf{B}} * \left(\mathrm{Diag}(\mathbf{p}^{\mathsf{UL}})\cdot\overline{\mathbf{A}}\right)\right]^T\cdot\mathbf{h}\right\}. \quad (29)$$

As a remark, we note that the Jacobian matrix in the logarithmic model has been conveniently expressed using variables in the linear model. Thus, $\mathbf{J}_{\underline{\phi}^{\mathsf{UL}}}(\overline{\underline{\mathbf{h}}})$ is simply given by replacing $\mathbf{h}$ with $\overline{\mathbf{h}}$ in (28) and (29).

Similarly, for the downlink, we can write

$$\underline{\phi}^{\mathsf{DL}} \approx \underline{\phi}^{\mathsf{DL}}(\overline{\underline{\mathbf{h}}}) + \mathbf{J}_{\underline{\phi}^{\mathsf{DL}}}(\overline{\underline{\mathbf{h}}})\cdot(\underline{\mathbf{h}}-\overline{\underline{\mathbf{h}}}) + \underline{\mathbf{n}}^{\mathsf{DLI}} \quad (30)$$

with $\underline{\mathbf{n}}_{\mathsf{DLI}} \sim \mathcal{N}(0, \sigma^2_{\mathsf{DLI}}\mathbf{I})$. The Jacobian turns out to be

$$\mathbf{J}_{\underline{\phi}^{\mathsf{DL}}}(\underline{\mathbf{h}}) = (\mathbf{Z}^{\mathsf{DL}})^{-1}\cdot\left[\left(\mathrm{Diag}(\mathbf{p}^{\mathsf{DL}})\cdot\overline{\mathbf{A}}^T\right) * \mathbf{I}_{K_\mathsf{U}}\right]^T\cdot\mathrm{Diag}(\mathbf{h}), \quad (31)$$

where

$$\mathbf{Z}^{\mathsf{DL}} \triangleq \mathrm{Diag}\left\{\left[\left(\mathrm{Diag}(\mathbf{p}^{\mathsf{DL}})\cdot\overline{\mathbf{A}}^T\right) * \mathbf{I}_{K_\mathsf{U}}\right]^T\cdot\mathbf{h}\right\}. \quad (32)$$

Then, after rearranging some terms in (20) and (30), the global observation model can be written as

$$\underbrace{\begin{bmatrix} \underline{\mathbf{r}} - \underline{\mathbf{p}} \\ \underline{\phi}^{\mathsf{UL}} - \underline{\phi}^{\mathsf{UL}}(\overline{\underline{\mathbf{h}}}) + \mathbf{J}_{\underline{\phi}^{\mathsf{UL}}}(\overline{\underline{\mathbf{h}}})\cdot\overline{\underline{\mathbf{h}}} \\ \underline{\phi}^{\mathsf{DL}} - \underline{\phi}^{\mathsf{DL}}(\overline{\underline{\mathbf{h}}}) + \mathbf{J}_{\underline{\phi}^{\mathsf{DL}}}(\overline{\underline{\mathbf{h}}})\cdot\overline{\underline{\mathbf{h}}} \end{bmatrix}}_{\triangleq\underline{\mathbf{y}}} \approx \underbrace{\begin{bmatrix} \mathbf{I}_K \\ \mathbf{J}_{\underline{\phi}^{\mathsf{UL}}}(\overline{\underline{\mathbf{h}}}) \\ \mathbf{J}_{\underline{\phi}^{\mathsf{DL}}}(\overline{\underline{\mathbf{h}}}) \end{bmatrix}}_{\triangleq\underline{\mathbf{W}}} \underline{\mathbf{h}} + \underbrace{\begin{bmatrix} \underline{\mathbf{n}}^{\mathsf{RSRP}} \\ \underline{\mathbf{n}}^{\mathsf{ULI}} \\ \underline{\mathbf{n}}^{\mathsf{DLI}} \end{bmatrix}}_{\triangleq\underline{\mathbf{n}}}, \quad (33)$$

where $\underline{\mathbf{n}} \sim \mathcal{N}(\mathbf{0}, \underline{\mathbf{D}})$ with

$$\underline{\mathbf{D}} \triangleq \mathrm{Diag}\left(\left[\sigma^2_{\mathsf{RSRP}}\mathbf{1}^T_K, \sigma^2_{\mathsf{ULI}}\mathbf{1}^T_{K_\mathsf{B}}, \sigma^2_{\mathsf{DLI}}\mathbf{1}^T_{K_\mathsf{U}}\right]^T\right). \quad (34)$$

Therefore, the LLMMSE estimator is given by

$$\hat{\underline{\mathbf{h}}}^{\mathsf{LLMMSE}} = \overline{\underline{\mathbf{h}}} + \mathbf{C}_{\underline{\mathbf{h}}}\underline{\mathbf{W}}^T(\underline{\mathbf{W}}\mathbf{C}_{\underline{\mathbf{h}}}\underline{\mathbf{W}}^T + \underline{\mathbf{D}})^{-1}(\underline{\mathbf{y}} - \underline{\mathbf{W}}\overline{\underline{\mathbf{h}}}). \quad (35)$$

Similarly, a *linearized log-LS (LLLS)* estimator can be obtained as follows,

$$\hat{\underline{\mathbf{h}}}^{\mathsf{LLLS}} = (\underline{\mathbf{W}}^T\underline{\mathbf{W}})^{-1}\underline{\mathbf{W}}^T\underline{\mathbf{y}}. \quad (36)$$

## V. Numerical results

For numerical evaluation, we adopt a simple but widely used correlation model known as the Kronecker model [11]. According to this model, the channel correlation matrix is the Kronecker product of the antenna correlation matrices at the transmit and receive side:

$$\mathbf{C}_{\mathbf{h}} = \rho\mathbf{R}_\mathsf{B}\otimes\mathbf{R}_\mathsf{U}\,,$$

where

$$[\mathbf{R}_\mathsf{B}]_{i,j} \triangleq \exp(-\alpha\|\mathbf{x}_{b(i)} - \mathbf{x}_{b(j)}\|^2)$$

and

$$[\mathbf{R}_\mathsf{U}]_{i,j} \triangleq \exp(-\beta\|\mathbf{x}_{u(i)} - \mathbf{x}_{u(j)}\|^2)\,.$$

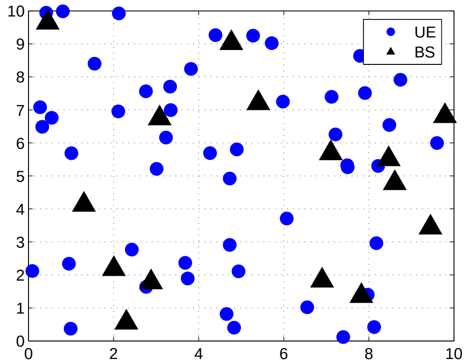



Fig. 1. Simulation scenario. 15 BSs and 50 users are randomly distributed in an area of square with a length of 10 km.

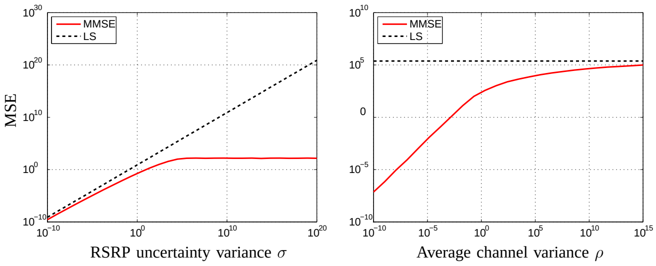
| Parameter | Value |
|---|---|
| $K_\mathsf{U}$ | 4 |
| $K_\mathsf{B}$ | 2 |
| $\alpha$ | $1/400$ km$^{-2}$ |
| $\beta$ | $1/300$ km$^{-2}$ |
| $\mathbf{p}^{\mathsf{UL}}$ | uniformly distributed in $[20, 23]$dB |
| $\mathbf{p}^{\mathsf{DL}}$ | uniformly distributed in $[43, 46]$dB |
| $\mathbf{p}$ | uniformly distributed in $[49, 51]$dB |
| $\rho$ | simulation variable |
| $\sigma^2_{\mathsf{RSRP}}$ | simulation variable |
| $\sigma^2_{\mathsf{ULI}}$ | $1.5\sigma^2_{\mathsf{RSRP}}$ |
| $\sigma^2_{\mathsf{DLI}}$ | $2\sigma^2_{\mathsf{RSRP}}$ |
| Path loss exponent | 2.5 |
| $\overline{\underline{\mathbf{h}}}$ | $\in[-126, -76]$dB |

TABLE I
SIMULATION PARAMETERS.

Here and hereafter, $\alpha$, $\beta$ are positive constants, $\|\cdot\|$ denotes Euclidean norm, and $\mathbf{x}_{b(i)}$, $\mathbf{x}_{u(i)}$ are the positions of the $i$-th BS and user $i$, respectively. Note that the Kronecker correlation model can be seen as a special case of the general model (4), with $d(\{b(i), u(i)\}, \{b(j), u(j)\}) = \alpha\|\mathbf{x}_{b(i)} - \mathbf{x}_{b(j)}\|^2 + \beta\|\mathbf{x}_{u(i)} - \mathbf{x}_{u(j)}\|^2$. Furthermore, $\rho$ is a simulation variable for evaluating the performance of our algorithm with different channel statistics. In addition the RSRP uncertainty $\sigma$ is used as a simulation variable to show the performance under different noise levels.

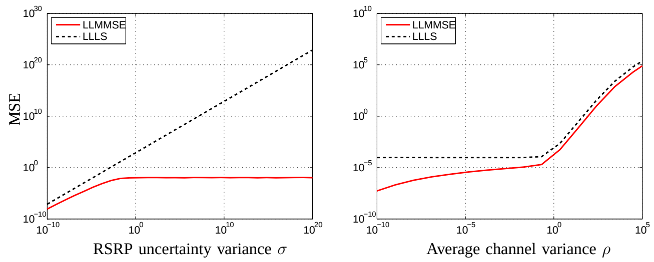
In the simulation, 15 BSs and 50 users are randomly distributed in a square area with a length of 10 km. Users are assigned to the closest BS, while the other BSs are interfering nodes in the downlink and interfered by the other users in the uplink. An example of the simulation topology is depicted in Fig. 1, where the path loss ranges between $-126$ dB and $-76$ dB. The other values used in the simulation are listed in Table I.

Numerical results are shown in Fig. 2 and 3, respectively, for the linear and the logarithmic model. We compare the MSE obtained by the (LL)MMSE and (LL)LS estimators as a function of the RSRP uncertainty variance $\sigma^2_{\mathsf{RSRP}}$ and of the channel variance $\rho$. Note that, in the logarithmic model,




(a) (b)

Fig. 2. MMSE and LS performance with the linear model (i.e. both the channel gain vector and the vector of measurement uncertainties are normally distributed): (a) vs. $\sigma^2_{\mathsf{RSRP}}$, with $\rho = 0.2$; (b) vs. $\rho$, with $\sigma^2_{\mathsf{RSRP}} = 3 \cdot 10^4$.




(a) (b)

Fig. 3. MMSE and LS performance with the logarithmic model i.e. both the channel gain vector and the vector of measurement uncertainties have log-normal distributions): (a) vs. $\sigma^2_{\mathsf{RSRP}}$, with $\rho = 1.5 \cdot 10^{-4}$ dB; (b) vs. $\rho$, with $\sigma^2_{\mathsf{RSRP}} = 1.3 \cdot 10^{-5}$ dB.

such variances apply to values in dB. The results show that in both cases the MMSE estimators behave according to the theory and outperform LS estimators. The gap vanishes for low noise uncertainty (Fig. 2 (a) and Fig. 3 (a)) because with sufficiently small diagonal elements of $\mathbf{D}$, the solution of (LL)MMSE converges to the solution of (LL)LS. The same conclusion can be drawn for the case with high channel variance (Fig. 2 (b) and Fig. 3 (b)): if $\mathbf{C}_{\mathbf{h}}$ is very large, we can neglect $\mathbf{D}$ so that (LL)MMSE and (LL)LS yield the same solution. Figure 3 shows the performance is still consistent when linearization is applied, except for very high channel variance where the approximation by the first-order Taylor expansion in (20) becomes inaccurate.

## VI. Numerical Complexity

From the numerical point of view, we remark that the linear system (under both models) is well conditioned as long as the number of available measurements is $\geq K$, so that matrix $\mathbf{W}$ (or $\underline{\mathbf{W}}$) is "tall". Since the number of RSRP measurements is $K$, such requirement is always satisfied if RSRP measurements are available for all BS-UE pairs.

The numerical complexity is dominated by the inversion of a matrix of size $(K + K_{\mathsf{B}} + K_{\mathsf{U}}) \times (K + K_{\mathsf{B}} + K_{\mathsf{U}})$, which has complexity $O((K + K_{\mathsf{B}} + K_{\mathsf{U}})^3)$. Note that in practice decomposition methods (e.g. QR factorization) are used for the solutions of linear systems without explicitly inverting the matrix. A cubic complexity in the number of measurements may be affordable if interference estimation is performed in a centralized way. In future research, more efficient distributed methods will be investigated for a possible decentralized implementation.

## VII. Conclusions

We have proposed two MMSE estimators for interference identification in LTE networks, assuming first an idealized linear Gaussian model and then a more realistic log-normal model. Both estimators are expressed in closed form. The second estimator achieves good performance provided that the channel fluctuations are not too far from the prior mean. Thanks to the logarithmic formulation, it is robust against numerical problems (overflow/underflow) and is suitable for practical implementation. Compared to iterative approaches, e.g. [8], the proposed solution enables real-time estimation of the interference coefficients and exploits the available prior information, namely spatial channel correlation and measurement uncertainty statistics.

The proposed approach can be extended to the case of sequential estimation (an extended Kalman filter can be derived using the same linearization as in Section IV). Distributed implementation may be investigated as well, exploiting the problem sparsity (every measurement only depends on a subset of elements of $\mathbf{h}$).

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