Reliability Modeling and Analysis of a Wireless Transmission as a Repairable system

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***Abstract*—Reliability of wireless transmission has been identified as an instrumental property in future 5G wireless systems due to the service expansion of 3GPP networks into domains such as Automotive and Industry. *Apriori* indication of the availability of the transmission link would enable oppurtunistic scheduling of safety critical services/applications when the link conditions are fair enough. Traditional reliability engineering methods such as Reliability Block Diagrams (RBD'S), life distributions and fault trees etc. can be used to predict the link availability. However, these methods can only be applied to binary states (up and down) and are more suitable for non-repairable systems only. This paper introduces the reliability modeling and analysis of the wireless transmission by considering the wireless transmission as a renewal process with arbitrary failure(λ) and repair(μ) rates. Reliable service composition (graceful degradation of the service's reliability requirements) has also been introduced by employing multi-state markov chains. To this end, this paper also introduces two new KPI's - Point Availability and Interval Availability for the reliability analysis of the transmission.**

I. Introduction

Ultra-Reliable Communication (URC) has been seen as one of the key enabler for future 5G mobile networks facilitating services/applications from automotive and industrial domains to be integrated into the 3GPP based networks. However, reliability in wireless networks is a relatively new research area compared to the wired networks, and in particular, wired network survivability. In general, reliabilty in wireless networks is harder to achieve owing to the time dependent factors on the effective links such as fading, interference, hardware failures etc. Techniques such as link adaptation by power control, Modulation and Coding Schemes (MCS) and retransmissions exploit the time diversity of the wireless transmission to ensure the success probability of the packet delivery before a certain deadline.

Hence, the transmission can be considered as a renewal process with failure (λ) and repair (μ) rates where the failure of the transmission is attributed to factors such as fading, interference etc and repair (restoration) due to the reliability techniques (e.g.,Link adaptation). Reliability analysis techniques such as Continuos Markov Chains and partial differential equations can be used to predict the Point Availability (PA) of any transmission instance [1]. Additionally, these techniques also support include Reliable Service Composition i.e., graceful degradation of service/application instead of making a binary decision of service available/not available [2]. From the link level point of view, the main idea behind reliable service composition is to set the framework for building protocols and transmission techniques that can adaptively switch to a lower grade of service (minimal connectivity) rather than failing the connection.

The rest of the paper is organized as follows. Section II presents the mathematical interpretation of transmission as a renewal process and the formulae for calculating Point and Interval availability (PA and IA). Section III

II. Theoretical Background

In order to analyze the reliability of a repairable system, consider a transmission instance between two peers. The reliability of the transmission mainly depends on the reliability of the instantaneous wireless link between the two peers and exhibits diversity with respect to space, time and frequency. This can be modeled as a renewal process with interarrival probabilities of failure and repair rates.

A. Transmission as a Renewal Process

Let us assume that the target reliability R_{tar} for a transmission is achieved by careful variation of techniques against factors at discrete time points $\tau_0, \tau_1, ...$ To define a renewable process, let $\tau_0, \tau_1, ...$ be stochastically independent and non-negative random variables distributed according to

$$F(x) = Pr(\tau_i \leqslant x), i = 1, 2, ..., x \geqslant 0.$$

The random variables

$$s_n = \sum_{i=0}^{n-1} \tau_i, n = 1, 2...,$$

or equivalently the sequence $\tau_0, \tau_1, ...$ constitutes a renewal process. The points $s_1, s_2, ...$ are renewal points (regeneration points) which in our case are the points where restoration of the faltering transmission takes place over the time. The renewal process can be associated with a *count function*

$$v(t) = \begin{cases} 0 & \text{for } t < \tau_0 \\ n & \text{for } s_n \leqslant t \leqslant s_{n-1} \end{cases}$$

giving the number of renewal points in the interval $(0, t]$. As $\tau_0 > 0, \tau_1 > 0, ..$ are *interarrival times*, the variable x starting by 0 at $t = 0$ and at each renewal point $s_1, s_2, ...$ (arrival times) is used instead of t as shown in Figure 1.

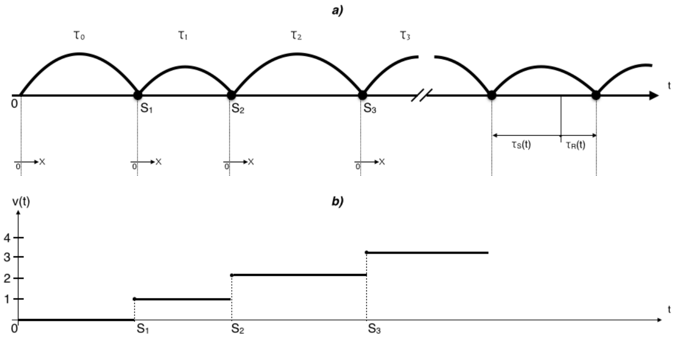

Fig. 1. a) Possible time schedule for a renewal process; b) Corresponding count function $v(t)$

B. Renewal Function, Renewal Density

Consider first the distribution function of the number of renewal points $v(t)$ in the time interval $(0, t]$. From Figure 1

$$
\begin{aligned}
Pr\left\{v(t) \leqslant n-1)\right\} &= Pr\left\{s_n > t\right\} = 1 - Pr\left\{s_n \leqslant t\right\} \\
&= 1 - Pr\left\{\tau_0 + ... + \tau_{n-1} \leqslant t\right\} \qquad (1) \\
&= 1 - F_n(t),\ n = 1, 2, ...
\end{aligned}
$$

The functions $F_n(t)$ can be recursively calculated as $F_1(t) = F_A(t)$,

$$F_{n+1}(t) = \int_0^t F_n(t-x) f(x) dx,\ n = 1, 2, ... \qquad (2)$$

From (1), it follows that

$$
\begin{aligned}
&Pr\left\{v(t) = n)\right\} = Pr\left\{v(t) \leqslant n)\right\} - Pr\left\{v(t) \leqslant n-1\right\} \\
&= F_n(t) - F_{n+1(t)}),\ n = 1, 2, ...
\end{aligned}
$$

and thus, the expected value (mean) of $v(t)$,

$$E[v(t)] = \sum_{n=1}^{\infty} n[F_n(t) - F_{n-1}(t)] = \sum_{n=1}^{\infty} F_n(t) = H(t) \quad (3)$$

The function $H(t)$ (Eq.3) is the *renewal function* and the function

$$h(t) = \frac{dH(t)}{dt} = \sum_{n=1}^{\infty} f_n(t) \qquad (4)$$

is the *renewal density*. Eq 3 and 4 are known as renewal equations and have exactly a solution whose Laplace transforms exist [3]. Thus, an ordinary renewal process is completely characterized by its renewal density $h_o(t)$ or renewal function $H_o(t)$

C. Repairability

The repairability can be introduced into the wireless transmission by considering reliability factors such as channel and interference as contributing to failures of the transmission and reliability techniques (link adaptation,retransmissions) contributing to the repair of the transmission. For simplification, let us assume that the transmission is restored to *as good as new condition* which means that the transmission is resumed with same availability as before once the repair is finished. It is also assumed that only one repair operation is possible at a time and the transmission undergoes no further failures during *maintenance*. However, these assumptions will also be considered later when we deal with semi-regenerative processes. The repairable transmission is completely characterized by the distribution function of the *failure-free times* $\tau_0, \tau_1, ...$ and the *repair times* $\tau_0', \tau_1', ...$ whose distributions (denoted as $F(x)$ and $G(x)$ can be given based on Eq.1 [1]

$$
\begin{aligned}
F(x) &= Pr(\tau_0 \leqslant x)\ and \\
G(x) &= Pr(\tau_0' \leqslant x) \qquad (5)
\end{aligned}
$$

with densities

$$f(x) = \frac{dF(x)}{dx}\ and\ g(x) = \frac{dG(x)}{dx} \qquad (6)$$

This can be modeled via the alternating renewal process which alternate from one state to the other after a stay (sojourn) time distributed according to $F(x)$ and $G(x)$ respectively. The two states will be referred to as the *up state* and *down state*, abbreviated as u and d respectively.Alternative renewal process is nothing but the generalization of the renewal process $F(x)$ as discussed in the previous section by introducing a positive random replacement time, distributed according to $G(x)$ [1].

To describe the generalization, let us introduce the two-dimensional stochastic process $(\zeta(t), \tau_{R\zeta(t)}(t))$ where $\zeta(t)$ denotes the state of the transmission.

$$\zeta(t) = \begin{cases} u & \text{if the transmission is up at } t \\ d & \text{if the transmission is dowm at time } t \end{cases}$$

$\tau_{Ru}(t)$ and $\tau_{Rd}(t)$ are thus the forward recurrence times in the up and down states respectively, provided that the item is up of down at time t.

To investigate the general case, considering factors and techniques as *two independent* renewal processes (τ_i) and (τ_i'), $i = 0, 1, ...$ In the case of transmission reliability, τ_i denotes the i^{th} *failure-free time* and τ_i' the i^{th} *repair* (restoration) time.

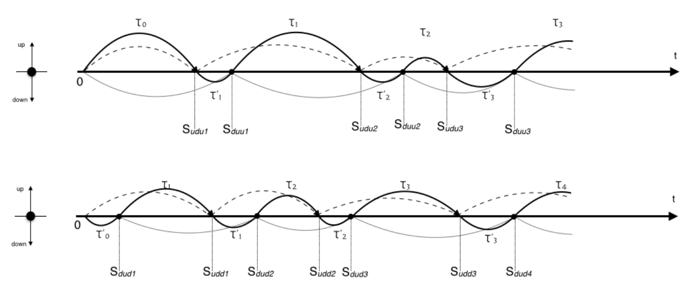

Fig. 2. Possible time schedule for factors and techniques starting at $t = 0$ with τ_0 and τ_1' respectively

Figure 2 shows a possible time schedule of these two alternating renewal processes. Embedded in every one of these processes are two renewal processes with renewal points S_{udui} or S_{uddi} marked with ▲ and S_{duui} or S_{dudi} marked with •

where *udu* denotes a transition *from up to down given up at t=0*, i.e.,

$$S_{udu_1} = \tau_0 + ({\tau_1}' + \tau_1) + ... + (\tau_{i-1}' + \tau_{i-1})$$

Both the the alternating renewal processes of Figure 2 must be combined to calculate the *availability* of the transmission. For this, let

$$p = Pr\left\{transmission\ up\ at\ t = 0\right\} \quad and$$
$$1 - p = Pr\left\{transmission\ down\ at\ t = 0\right\}$$

In terms of the process $(\zeta(t), \tau_{R\zeta(t)}(t))$,

$$p = Pr\left\{\zeta(0) = u\right\}, \qquad F_A(x) = Pr\left\{\tau_{Ru} \leqslant x \mid \zeta(0) = u\right\}, \tag{7}$$

$$1 - p = r\left\{\zeta(0) = d\right\}, \qquad G_A(x) = Pr\left\{\tau_{Ru} \leqslant x \mid \zeta(0) = d\right\}. \tag{8}$$

Consecutinve jumps from *up* to *down* form a renewal process with renewal density

$$h_{ud}(t) = ph_{udu}(t) + (1-p)h_{udd}(t) \tag{9}$$

Similarly, the renewal density for consecutive jumps from *down* to *up* is given by

$$h_{du}(t) = ph_{duu}(t) + (1-p)h_{dud}(t) \tag{10}$$

Using Equations (7) and (8)

$$\begin{aligned} &Pr\left\{\zeta(t) = u \cap \zeta_{Ru}(t) > \theta\right\} \\ &= p(1 - F_A(t+\theta)) + \int_0^t h_{du}(x)(1 - F(t - x + \theta))dx \end{aligned} \tag{11}$$

and

$$\begin{aligned} &Pr\left\{\zeta(t) = d \cap \zeta_{Rd}(t) > \theta\right\} \\ &= (1-p)(1 - G_A(t+\theta)) + \int_0^t h_{ud}(x)(1 - G(t - x + \theta)) \\ &dx \end{aligned} \tag{12}$$

Setting $\theta = 0$ in Eq.(9) yields

$$Pr\left\{\zeta(t) = u\right\} = p(1 - F_A(t)) + \int_0^t h_{du}(x)(1 - F(t - x))dx \tag{13}$$

The Probability $PA(t) = Pr\left\{\zeta(t) = u\right\}$ is called the point availability and gives the probability of the transmission to be operating accordingly at time t.

$$PA(t) = Pr\left\{\text{up at t} \mid \text{new at t=0}\right\}$$

and $IA(t, t+\theta) = Pr\left\{\zeta(t) = u \cap \zeta_{Ru}(t) > \theta\right\}$ is the interval reliability that gives the probability that transmission operates failure free during an interval $[t, t+\theta]$

$$IA(t) = Pr\left\{\text{up at } [t, t+\theta] \mid \text{new at t=0}\right\}$$

Other kinds of availability such Mission Availability (MA), Work-Mission Availability (WMA) and Joint Availability (JA) are also useful for other applications but are not considered here.

III. Reliability Analysis

Since reliability analysis techniques such as RBD's and life distributions are not suitable for multi-state repairable systems, Markov processes [4] are used which represent a straightforward generalization of sequences of independent random variables. Markov processes are processes without *after effect* i.e., the evolution of the process after an arbitrary time point t only depends on t and on the state occupied at t, not on the evolution of the process before t. For time-homogeneous Markov processes, the dependence on t also disappears (memory-less property). Markov processes are very simple regenerative stochastic processes. They are regenerative with respect to each state and, if time-homogeneous, also with respect to any time t. Semi-Markov processes have the Markov property at the time points of any state change; i.e., all states of a Semi-Markov process are regeneration states. In a semi-regenerative process, a subset $Z_0, ...Z_k$ of the states $Z_0, ...Z_m$ are regeneration states and constitute an embedded semi-Markov process. Typically, regeneration points occur when the process returns to some particular states (regenerative states). Between regeneration points, the dependency structure of the process can be very complicated.

Semi-Markov Process for Transmission Reliability Analysis

The idea of a semi-Markov process was proposed in 1954 [5]. The semi-Markov process is similar to the Markov process is that both processes are described by a set of states $Z_1, Z_2, ..., Z_n$ whose transitions are governed by a transition probability matrix T. The stay time in a given state Z_i is a random variable $\tau_{ij} > 0$ whose distribution depends on Z_i and on the following state Z_j.
To define a semi-Markov process, let $\xi_0, \xi_1, ...$ be the sequence of *consecutively* occuring states i.e., the sequence of random variables taking values in $Z_0, Z_1, ..., Z_m$, and $\eta_0, \eta_1, ...$, the stay times between consecutive states. A stochastic process $\xi(t)$ with state space $Z_0, Z_1, .., Z_m$ is a semi-Markov process with finite number of states if for $n = 0, 1, 2, ...$, arbitrary $i, j, i_0, ...i_{n-1} \in \{0, ..., m\}$, and arbitrary $x_0, ..., x_{n-1} > 0$

$$\begin{aligned} &Pr\left\{(\xi_{n+1} = Z_j \cap n_n < x) \mid (\xi_n = Z_i \cap n_{n-1} \right. \\ &= x_{n-1} \cap ...\xi_1 = Z_{i_1} \cap \eta_0 = x_0 \cap \xi_0 = Z_{i_0}) \left.\right\} \\ &= Pr\left\{(\xi_{n+1} = Z_j \cap \eta_n \leqslant x) \mid \xi_n = Z_i\right\} = Q_{ij}(x) \end{aligned} \tag{14}$$

The functions $Q_{ij}(x)$ in Eq.12 defined for $j \neq i$, are the semi-Markov transition probabilities, also known as one-step transition probabilities and all the states $Z_0, Z_1, .., Z_m$ are regeneration states. The Point Availability (PA) i.e., the probability that the transmission is in one of the up states at t, given it was in Z_i at $t = 0$ and the reliability function R(t) which is the probability that the first transition from a UP state to a DOWN state occurs after time t can be calculated by means of differential and integral equations respectively [1].

Reliable Service Composition

Reliable Service Composition (RSC),i.e., the graceful degradation of the service reliability requirements has been introduced in [2] inorder to guarantee a certain reliability of the transmission service parameters. The main idea behind RSC is to set the framework for building protocols and transmission techniques that can adaptively switch to a lower grade of service (minimal connectivity) rather than failing the connection. If the reliability requirement is stated rigidly, e.g. 'transfer of data packets of size B bytes under L seconds in 99% of the attempts', then there is a clear criterion/constraint to decide whether the reliability requirement is met or not. On the other hand, the reliability of the wireless link is defined through its Outage Rate Probability (ORP) as done in [6], which is the probability that the system cannot reach a certain data-rate at a given QoS. It is also crucial to note that, when the system is in outage, it supports a lower data rate at a certain QoS/error probability. Hence, instead of making a binary decision "service available/not available", we can also look into the composition of the service and define composite QoS/QoE, such as one of more degraded variants of the same service. For example, in a V2V or V2I scenario, if a congestion is detected, then the system may decide to send only the critical messages or shorten their size. In this case, the Availability Indication should also accomodate soft values between 0 and 1 and this is where RBD's fail and markov models succeed.

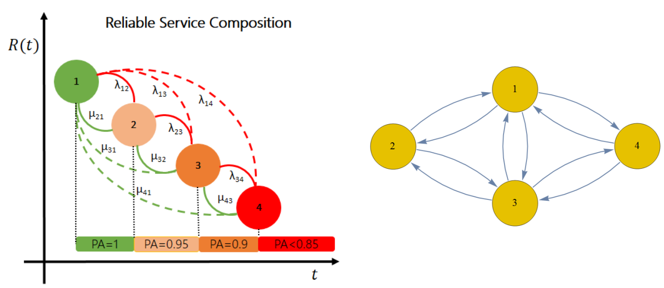

Fig. 3. RSC and the corresponding Markov Graph

Figure 3 shows an example RSC with 4 compositions represented by a multi decrement state model and the corresponding Markov graph. State 1 is assumed to be the initial state ($t = 0$) and states 2 and 3 are degraded states with lower service requirements. State 4 is the DOWN state also referred to as the absorption state in the Markov model.

IV. System Model

Let us consider a mobile UE connected to its peer and requested a URC link by means of an Availability Request(AR) as explained in [7]. In order to support RSC, the AR should comprise the reliability requirements in different service configurations (e.g., required availability for supported degraded operation states). The Availability Indication can be either a soft value between 0 or 1 or the similar binary indications for the varied configurations. Continous time markov states with different reliability requirements were used to represent the service configurations.
The life distributions of pathloss, shadowing and multi-path fading from [8] were used to calculate the failure times (hazard functions) of different states (e.g., $\lambda_{12}, \lambda_{13}$ denoting the hazard functions or in other terms, conditional probabilities).
The renewal points (S) considered in our simulation were 100. The corresponding states and their values haven been calculated as the point reliability (survival function) of the transmission [8]. This results in experimental data generation of the reliability service classes. The reliability function $R(t)$ along with the hazard function $\lambda(t)$ as a function of time (x-axis) of the transmission is given in figure 4.

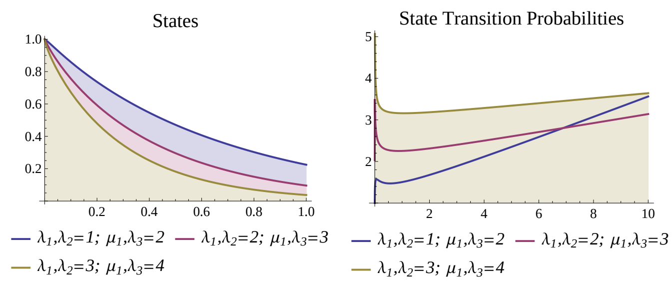

Fig. 4. $R(t)$ and $\lambda(t)$ for calculating the states (s) and transition matrix (T)

The red, blue and green curves represent different random values of the mean and standard deviations of the considered life distributions. The state transition matrix T was constructed by calculating the state transition probabilities between consecutive (e.g.,$\lambda_{12}, \lambda_{23}...\lambda_{n-1,n}$ and non-consecutive (e.g.,$\lambda_{13}, \lambda_{25}...\lambda_{n,m(m\subset n)}$ states of the transmission. The repair rates (i.e., $\lambda_{31}, \lambda_{21}...\lambda_{n,m(m>n}$ were assumed to be constant (0.2). A detailed investigation of the repair rate modeling will be investigated in the future work.

Reliability Analysis

The important quantities for the reliability analysis using Markov processes are the *state probabilities* obtained from T and the distribution function of the stay (sojourn) times in the set of system UP states. The state probabilities allow calculation of the PA. The reliability function R(t) can be obtained from the distribution function of the stay (sojourn) times in the set of system *up* states as explained in Section II.C and III.
In order to derive the PA and R(t), it is required to differentiate between the system of UP states and DOWN states. Let us consider three applications with varying service reliability configurations as explained in III - Low, Medium and High as shown in table I. A Markov model is constructed with 100 uniformly distributed states (100 is the absorption state).

Point Availability (PA): PA can also be defined as the probability that the transmission is in the set of UP states at time t [1]. Figure 5 shows the PA of the three service configurations over time. It can be seen that the PA is 1 for all service variants due to the assumption that the transmission starts anew. The transmission reliability continuously decreases over time and this behavior can be attributed to the fact that the repair rates

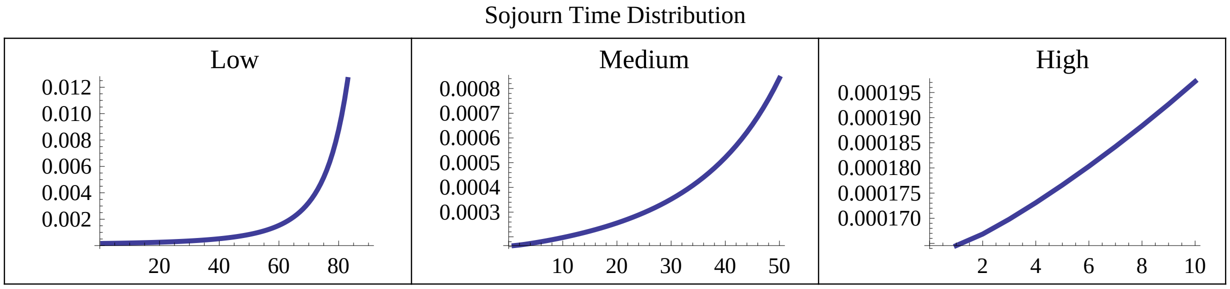

Fig. 5. Sojourn time distributions of UP states in 3 Service Variants

TABLE I
RELIABILITY SERVICE CLASSES

Service	Reliability	UP States	DOWN States
1	Low	1-90	91-100
2	Medium	1-50	51-100
3	High	1-10	11-100

were assumed to be constant at 0.2. It can be seen that the PA of medium and high reliability services is almost similar unless seen at a higher time granularity.

$$[PA = Pr\,\{\text{Transmission is in UP state at time t}\}$$
$$= 1 - Pr\,\{\text{Transmission is in DOWN state at time t}\}$$
$$= 1 - CDF\,\{\text{First Hitting Time of DOWN states}\}$$

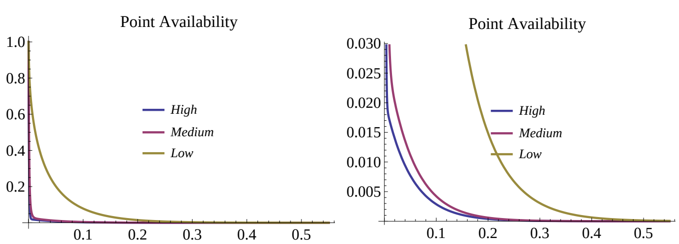

Fig. 6. PA

Reliability Function R(t): The Reliability function R(t) is a mathematical expression analytically relating the probability of successful transmission to time [8]. In the case of repairable system analysis using markov chains, R(t) can be obtained from the distribution function of the stay time in the set of system UP states [1] as shown in Figure 5. The x-axis correspond to the UP states and y-axis correspond to the proportion of time spent in each of the state. It is clearly visible that lower reliability states are highly absorbing ($\lambda > \mu$) repair rate). This is once again due to the assumption of a constant μ.

V. CONCLUSIONS AND FUTURE WORK

Availability Indication is a key enabler for ultra-reliable communications in 5G due to the safety criticality of the applications/services involved. This paper introduced a novel availability prediction and reliability analysis of a wireless transmission assuming it as a repairable system with variable failure and repair rates. Using the standard markov chains, we were able to derive the initial values of a link availability over time considering the standard channel models. This methodology can be further extended to bring in all the failure and repair factors in order to predict the link availability more precisely. Since the applicable domains for this framework are automotive and industrial applications, energy and computation capability will not be a bottleneck. To this end, the concept of Reliable Service Composition (RSC) is also introduced defining composite QoS/QoE with more than one degraded variants of the same service. As future work, we intend to make the framework more detailed and also consider realistic repair rate values.

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ACKNOWLEDGEMENTS

Part of this work has been performed in the framework of the FP7 project ICT-317669 METIS, which is partly funded by the European Union. The authors would like to acknowledge the contributions of their colleagues in METIS, although the views expressed are those of the authors and do not necessarily represent the project.