Practical Joint Network-Channel Coding Schemes for Orthogonal Multiple-Access Multiple-Relay Channel

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***Abstract*—In this paper, we investigate the benefit of Joint Network Channel (JNC) coding and decoding for the slow fading Multiple Access Multiple Relay Channel (MAMRC), defined as follows: (1) Independent sources communicate with a single destination in the presence of multiple relays; (2) Each relay is half-duplex and applies a Selective Decode and Forward (SDF) relaying strategy, i.e, it forwards a deterministic function of the error-free decoded messages; (3) Orthogonal multiple access schemes are used by the sources and the relays; (4) The links between the different nodes of the MAMRC are subject to slow fading and additive white Gaussian noise. This model is referred to as Orthogonal MAMRC (OMAMRC). JNC coding and decoding framework fully exploits the spatial diversity and redundancy residing in both channel codes and network codes. The receivers of the relays and the destination are designed to benefit from the signals of the previously activated relays to better decode the sources. Simulations show the effectiveness of the proposed schemes.**

I. Introduction

Network coding, initially proposed by Ahlswede et al. in [1], is a powerful paradigm where intermediate nodes in a network are allowed not only to route but also to perform algebraic operations on the incoming data flows. The MAMRC, denoted by $(M, L, 1)$-MAMRC, consists of M independent users (or sources) which attempt to transmit their packets to a common destination with the help of L independent relays, where $M \geq 2$ and $L \geq 2$ are arbitrary integers. This channel model could be seen as a generalization of the relay channel, introduced by van der Meulen [2], and the multiple access relay channel, introduced by Kramer and Wijngaarden [3]. Unfortunately, the capacity of wireless $(M, L, 1)$-MAMRC in general is still not known [4].

In this paper, we retain a restrictive version of MAMRC, namely Orthogonal MAMRC (OMAMRC) with time-division based half duplex relays. A Selective Decode and Forward (SDF) cooperative (or relaying) strategy is adopted, where each relay tries to decode the packets of the sources and forwards a deterministic function of the correctly decoded ones (based on the checks of Cyclic Redundancy Check (CRC) codes inserted in the sources' packets). SDF was first analyzed in a Separate Network Channel Coding/Decoding (SNCC/SNCD) framework from an information outage perspective in [5]. In SNCC/SNCD, the network coding/decoding is performed at the network layer and channel coding/decoding is performed at the physical layer separately where the channel codes are exclusively dedicated to turn the radio links into packet erasure channels. SDF was later extended to JNC coding/decoding (JNCC/JNCD) framework for semi-orthogonal and non-orthogonal multiple access relay channel in [6], [7]. The SDF has several advantages: (1) SDF prevents error propagation from the relays to the destination; (2) It reduces the energy consumption at the relays; (3) The sources who have poor source-to-relay (S-R) links, with high probability, will not prevent the relays from helping the other sources. The derivation of the individual and common outage probabilities of various access schemes of $(M, L, 1)$-MAMRC, when SDF relaying and JNCC/JNCD framework is used, was presented in [8]. In practice, network codes are linear and can be implemented using two main approaches, either Binary Field Network Coding (BFNC) or Galois Field (with 2^q elements, $q \geq 2$) Network Coding (GFNC). Deterministic BFNC and GFNC schemes that achieve the full diversity of $(M, L, 1)$-OMAMRC, with single antenna links, are presented in [9], and [10], respectively. In [11], the authors propose a practical JNCC/JNCD scheme for $(M, L, 1)$-OMAMRC by combining non-binary irregular low-density parity-check channel coding and random GFNC. Perfect (S-R) links were assumed and a sub-optimal Soft-In Soft-Out (SISO) network decoder was proposed, namely *selection updating decoder*.

The main contributions of this paper are the following: (1) Practical JNCC/JNCD schemes are proposed for the slow fading $(M, L, 1)$-OMAMRC, with imperfect S-R links, since the assumption of error-free links is not realistic in wireless environments; (2) The listening/transmitting schedule of the relays is designed to provide each listening relay with an opportunity to benefit from the previously activated relays to better decode the sources' packets; (3) As far as network coding is concerned two different options are investigated and compared: A deterministic provably full diversity GFNC, and a special type of BFNC referred to as Bit Interleaved XOR (BI-XOR). For Channel coding, turbo codes are used to encode the sources' packets while punctured convolutional codes are used at the relays; (4) Finally, a factor graph based JNCD suitable for lossy S-R links and SDF is presented, which differs from the *selection updating decoder*.

The remainder of the paper is outlined as follows. In Section II, we introduce the system model. In Section III, we describe each aspect of our JNC distributed coding and decoding, including the relaying protocol. In Section IV, we compare different scenarios and demonstrate the efficiency of our approach. Some conclusions are drawn in Section V.

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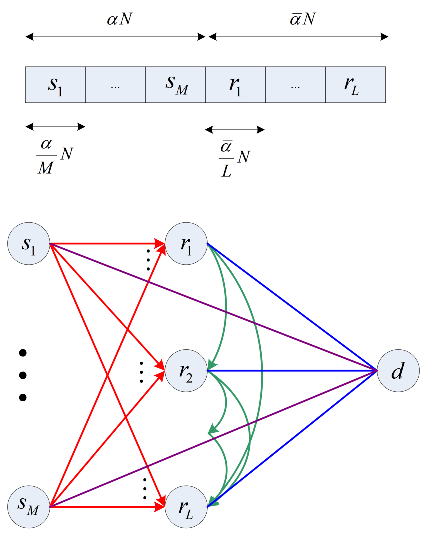

Fig. 1. $(M, L, 1)$-OMAMRC channel model and access scheme.

A. Notation

In the sequel, we use boldface small and capital letters to denote vectors and matrices, respectively. Let $\mathbf{A}$ be a matrix with i^{th} row $\mathbf{a}^i$ and j^{th} column $\mathbf{a}_j$, entry (i,j) is denoted $a_{i,j}$ or equivalently $[\mathbf{A}]_{i,j}$. The n-square identity matrix is denoted $\mathbf{I}_n$. $\mathbf{0}_n$ and $\mathbf{1}_n$ stand for n-tuples of zeros and ones, respectively. Let $x_{\mathcal{S}}$ denote the vector $[x_{s_1}, \dots, x_{s_{|\mathcal{S}|}}]^\top$, where $^\top$ is the transpose operator. Similarly, we define the block vector $\underline{\mathbf{x}}_{\mathcal{S}}$ as $[\mathbf{x}_{s_1}^\top, \dots, \mathbf{x}_{s_{|\mathcal{S}|}}^\top]^\top$. $\mathbf{x} \sim p(\mathbf{x})$ means that the random vector $\mathbf{x}$ follows the probability distribution function (pdf) $p(\mathbf{x})$. $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is a circularly symmetric complex Gaussian random vector with mean $\mathbb{E}(\mathbf{x}) = \boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Calligraphic upper case letters are used to denote finite ordered sets of the form $\mathcal{S} = \{s_1, \dots, s_{|\mathcal{S}|}\}$ where $|\mathcal{S}|$ is the cardinality of the set $\mathcal{S}$. The empty set is denoted by $\{\emptyset\}$. The index of an element $s \in \mathcal{S}$ is denoted by $ind(s)$. Finite field (Galois filed) of order 2^q is denoted by $\mathbb{F}_{2^q}$. Given a condition C we define $\mathbb{1}_{\{C\}}$ as an indicator function, i.e., $\mathbb{1}_{\{C\}} = 1$ if C is true and $\mathbb{1}_{\{C\}} = 0$ otherwise.

II. System Model

Fig. 1 illustrates the $(M, L, 1)$-OMAMRC model and access scheme. We consider a set of statistically independent sources $\mathcal{S} = \{s_1, \dots, s_M\}$, each source $s \in \mathcal{S}$ wants to communicate its packet $\mathbf{u}_s \in \mathbb{F}_2^{K_s}$ of K_s information bits to a common destination d with the help of a set of relays $\mathcal{R} = \{r_1, \dots, r_L\}$. Each source s is equipped with one transmit antenna, each relay r is equipped with one transmit antenna and m_r receive antennas, where $m_r = m$, $\forall r \in \mathcal{R}$, and the destination is equipped with m_d receive antennas. All sources are symmetric, i.e., have the same transmission rate, power, priority, and $K_s = K$, $\forall s \in \mathcal{S}$. Perfect Channel State Information (CSI) is available only at the receiver of each direct link. The N available channel uses are divided into two successive time slots corresponding to the transmission phase of the sources (first phase), say $N_1 = \alpha N$ channel uses, and to the transmission phase of the relays (second phase), say $N_2 = \overline{\alpha} N$, where $\overline{\alpha} = 1 - \alpha$, with $\alpha \in]0, 1]$. In the first phase, each source is given a time slot of duration $N_s = \alpha N / M$ channel uses, and in the second phase, each relay is given a time slot of duration $N_r = \bar{\alpha} N / L$ channel uses (see fig. 1). We always assume that the number of channel uses defined are integers. We assume that the sources and the relays indices are ordered according to their time slots assignments in the first and second phase, respectively. Note that in OMAMRC, some of the sources, which are provided by the suitable relaying equipments, could work as relays in the second phase, i.e., $s_i = r_j$ for some $1 \leq i \leq M$ and $1 \leq j \leq L$. In general, we assume that $\mathcal{S} \cap \mathcal{R} \neq \{\emptyset\}$. During the second phase, each relay tries to decode the packets of the sources. Another interesting feature in OMAMRC is that the relays can listen to the previously activated relays hence, they can better decode the signals of the sources. Consequently, the listening period of relay r is $MN_s + (ind(r) - 1)N_r$ channel uses. Let $S_r \subseteq S$ denotes the decoding set of the relay r, i.e., the maximum set of sources that the relay r can decode without errors. $S_r = \{\emptyset\}$ means that the relay r remains idle in its time slot. Each relay generates a network coded packet, denoted by $\mathbf{u}_r \in \mathbb{F}_2^{K_r}$, of K_r information bits. We fix $K_s = K_r = K$, and define the individual spectral efficiency as $R = K/N$. During the time slot, assigned to a transmitting node $a \in \{s, r\}$, the received signal at node $b \in \{r, d\}$, where $a \neq b$, can be written as

$$\mathbf{Y}_{a,b,k} = \sqrt{\gamma_{a,b}}\mathbf{h}_{a,b} x_{a,k} + \mathbf{n}_{a,b,k}, \tag{1}$$

where $k = 1, \dots, N_a$, and $x_{a,k} \in \mathbb{C}$ is the transmitted symbol. The received samples form the matrix $\mathbf{Y}_{a,b} \in \mathbb{C}^{m_b \times N_a}$. The additive noise vectors $\mathbf{n}_{a,b,k}$ and the channel fading vectors $\mathbf{h}_{a,b}$ are independent identical distributed (i.i.d.) and follow $\mathcal{CN}(\mathbf{0}, \mathbf{I}_{m_b})$. The channel vectors $\mathbf{h}_{a,b}$ stay constant over N_a channel uses after which they switch to an independent values (slow fading). $\gamma_{a,b}$ is the average received power at the receiver b from transmitter a. $\mathbf{Y}_d$ denotes the received samples at the destination during N channel uses. $\mathbf{Y}_r$ is the received samples at the relays r during $MN_s + (ind(r) - 1)N_r$ channel uses

III. JNC Distributed Coding and Decoding

In this section, we explain our JNC distributed coding and decoding approach, detailing the structure of the encoders, when and how JNC decoding is performed.

A. Coding at the sources

The packets of the sources are binary vectors $\mathbf{u}_s \in \mathbb{F}_2^K$ of length K. Each source employs a Bit Interleaved Coded Modulation (BICM) [12]. Binary vectors are first encoded with linear binary codes $C_s : \mathbb{F}_2^K \longrightarrow \mathbb{F}_2^{n_s}$ into binary codewords $\mathbf{c}_s \in \mathbb{F}_2^{n_s}$. For C_s we choose punctured turbo codes of coding rate w_s. Each turbo code consists of two Recursive Systematic Convolutional (RSC) encoders with generator matrix $\mathbf{G}_s(D)$, concatenated in parallel using optimized semi-random interleaver π. Each binary codeword $\mathbf{c}_s$ is then bit-interleaved into $\mathbf{b}_s \in \mathbb{F}_2^{n_s}$ using π_s . Each interleaved binary codeword $\mathbf{b}_s$ is mapped by a memoryless modulator $\phi_s : \mathbb{F}_2^{q_s} \rightarrow \mathcal{X}_s$ to the modulated sequence $\mathbf{x}_s \in \mathcal{X}_s^{N_s}$.

B. Coding at the relays

At the end of the listening phase, each relay tries to decode the packets of the sources. To accomplish this, each relay uses the received samples from the sources and from the previously activated relays. The detailed description of the iterative decoder will be discussed in Sec. III-D. Based on CRC, a relay can decide if a packet is correctly decoded or not. When a relay $r \in \mathcal{R}$ decides to transmit (i.e., has correctly decoded at least one packet), it applies a local network coding function $F_r : \mathbb{F}_2^{K\times M} \rightarrow \mathbb{F}_2^K$ to generate its network coded packet. Let $\mathbf{u}_r = F_r(\mathbf{u}_{s_1}, \ldots, \mathbf{u}_{s_M})$ be the relay binary network coded packet of length K. Any incorrectly decoded source's packet will be replaced by the all-zeros vector $\mathbf{0}_K$. the detailed structure of F_r will be discussed in Sec. III-C. Each cooperating relay employs a BICM. Binary network coded packets are encoded with linear binary codes $C_r : \mathbb{F}_2^K \rightarrow \mathbb{F}_2^{n_r}$ into binary codewords $\mathbf{c}_r \in \mathbb{F}_2^{n_r}$. For C_r we choose punctured recursive systematic convolutional codes (RSCC) of coding rate w_r with generator matrix $\mathbf{G}_r(D)$, and puncturing pattern that gives more importance to the parity bits than the systematic bits. The binary codeword $\mathbf{c}_r$ is then bit-interleaved into $\mathbf{b}_r \in \mathbb{F}_2^{n_r}$ using π_r . The interleaved binary codeword $\mathbf{b}_r$ is mapped by a memoryless modulator $\phi_r : \mathbb{F}_2^{q_r} \rightarrow \mathcal{X}_r$ to the modulated sequence $\mathbf{x}_r \in \mathcal{X}_r^{N_r}$. The relation between the different parameters are

$$R = \left(\frac{M}{q_s w_s} + \frac{L}{q_r w_r}\right)^{-1}, \text{and } \alpha = \left(1 + \frac{L q_s w_s}{M q_r w_r}\right)^{-1}. \quad (2)$$

Note 1: To let the destination and the other relays detect which of the frames are included in a relay's transmitted signal, each relay transmits a side information (M additional bits) to indicate its state to the receivers. We assume the side information is received perfectly at the destination and the other listening relays.

Note 2: The separate functions F_r, which are applied locally at each relay, can be seen as components of a global function denoted by $F_{nc} : \mathbb{F}_2^{K\times M} \rightarrow \mathbb{F}_2^{K\times L}$, namely the *global network coding function*.

C. Network coding

In GFNC, the function F_r is traditionally seen as concatenation of three functions (1) $\psi : \mathbb{F}_2^K \rightarrow \mathbb{F}_{2^q}^{K/q}$ which converts a vector of bits of length K into a vector of $\mathbb{F}_{2^q}$ elements of length K/q ($q \geq 2$); (2) A linear function that combines the vectors of $\mathbb{F}_{2^q}$ elements after multiplying each one of them by a network coding coefficient; (3) ψ^{-1} which is the inverse function of ψ. Thus, the network coded packet of the relay $r \in \mathcal{R}$ can be written as

$$\mathbf{u}_r = F_r(\mathbf{u}_{s_1}, \ldots, \mathbf{u}_{s_M}) = \psi^{-1}\left(\sum_{s\in\mathcal{S}} \alpha_{s,r} * \psi(\mathbf{u}_s)\right), \quad (3)$$

where $\sum$ and $*$ are the summation and multiplication operations of $\mathbb{F}_{2^q}$, respectively. $\alpha_{s,r} \in \mathbb{F}_{2^q}$ are the network coding coefficients. Any incorrectly decoded packet in (3) will be replaced by the all-zeros vector $\mathbf{0}_K$ Using the properties of the Galois fields $\mathbb{F}_{2^q}$ [14], we can write (3) as binary operations of the form

$$\mathbf{u}_r = \sum_{s\in\mathcal{S}} \oplus \mathbf{G}_{s,r}\mathbf{u}_s, \quad (4)$$

where $\sum \oplus$ represents the summation in $\mathbb{F}_2$, and $\mathbf{G}_{s,r}$ is a $K \times K$ binary matrix depends on the chosen coefficients $\alpha_{s,r}$. The matrix $\mathbf{G}_{s,r}$ has the form

$$\mathbf{G}_{s,r} = \begin{bmatrix} \mathbf{G}(\alpha_{s,r}) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{G}(\alpha_{s,r}) \end{bmatrix}, \quad (5)$$

where $\mathbf{G}(\alpha_{s,r})$ is a $q \times q$ binary matrix, that corresponds to the multiplication operation by the network coding coefficient $\alpha_{s,r}$.

In BFNC, the network coded packet of a relay $r \in \mathcal{R}$ is given by

$$\mathbf{H}_r\mathbf{u}_r = \sum_{s\in\mathcal{S}} \oplus \mathbf{H}_{s,r}\mathbf{u}_s, \quad (6)$$

where $\mathbf{H}_r$, and $\mathbf{H}_{s,r}$, for all $s \in \mathcal{S}$, and $r \in \mathcal{R}$ are $K \times K$ binary matrices. From (4) and (6), a direct relation between the BFNC and the GFNC is seen, both coding schemes are similar if $\mathbf{H}_r = \mathbf{I}_K$ and $\mathbf{H}_{s,r} = \mathbf{G}_{s,r}$. Thus, code designs for GFNC can be directly translated to code designs for BFNC, but the converse is not true in general.

Two network coding strategies at the relays are considered:
1) GFNC with coefficients chosen from Maximum Distance Separable (MDS) codes and randomly interleaved packets. Each relay r generates its network coded packet using the following operation

$$\tilde{\mathbf{u}}_r = \sum_{s\in\mathcal{S}} \oplus \mathbf{G}_{s,r}\tilde{\mathbf{u}}_{s,r}, \quad (7)$$

where $\tilde{\mathbf{u}}_{s,r} = \Pi_{s,r}(\mathbf{u}_s)$, and $\tilde{\mathbf{u}}_r = \Pi_r(\mathbf{u}_r)$ are interleaved versions of the packets of the sources and the relay network coded packet, respectively. The interleavers $\Pi_{s,r}, \Pi_r$ are independent pseudo random interleavers of length K. They are used to insure independence between the circulated messages and overcome the effect of short cycles, which result from the dense MDS network coding matrix, in the message passing decoding algorithm. For $(2, 2, 1)$-OMAMRC, we choose the network coding coefficients from $\mathbb{F}_4$ with generator polynomial $\alpha^2 + \alpha + 1$. $\alpha_{s_1,r_1} = \alpha_{s_2,r_1} = 1$, $\alpha_{s_1,r_2} = 1$, and $\alpha_{s_2,r_2} = \alpha$. For a general $(M, L, 1)$-MAMRC, we use [M+L,M,L+1] Reed-Solomon codes.

2) BI-XOR, initially proposed in [15], is a special type of BFNC where $\mathbf{H}_r$, and $\mathbf{H}_{s,r}$ are independent pseudo-random interleavers, and represented by Π_r, and $\Pi_{s,r}$, respectively. This network coding scheme has the advantages of random network codes, and is simple at the same time. Furthermore, BI-XOR can achieve a diversity order close to full diversity with high probability [15, Theorem 5]. Each relay r generates its network coded packet using (7) by replacing $\mathbf{G}_{s,r}$ by $\mathbf{I}_K$.

D. JNC decoding at the destination and the relays

The destination and the relays use the same decoding principle which relies on JNCD. The destination evokes its decoding process at the end of the second phase by processing the received samples $\mathbf{Y}_d$, (see (1)) and taking into consideration the cooperation modes (side informations) of the L relays. Each relay starts its decoding process at the beginning of its time slot by processing the received samples $\mathbf{Y}_r$ and taking into consideration the cooperation modes of the $ind(r)-1$ relays (we assume that the processing time of JNCD at the relays is fast and will not cause a delay). In this section, we describe the decoding

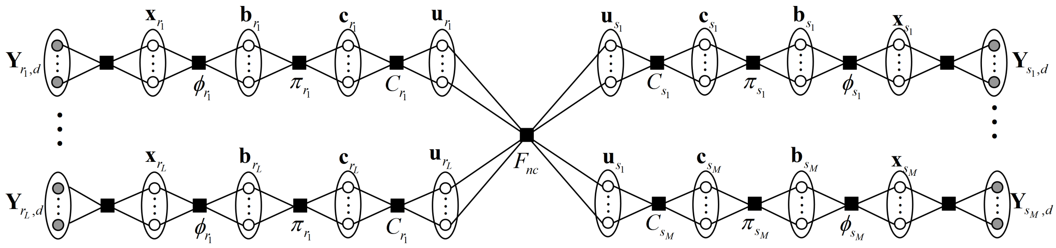

Fig. 2. Factor graph of $(M, L, 1)$-OMAMRC

process of the destination for the full cooperation mode, i.e., all the relays have correctly decoded all the frames of the sources, the other cooperation modes are obtained by removing (deactivating) some elements from the full cooperation decoder. Consider the behavioral model of $(M, L, 1)$-OMAMRC at the destination is denoted by B, which captures the relationship between the different variables of the system. The characteristic function for B, denoted by Ξ_B, can be factorizes into many sub characteristic functions as

$$\begin{aligned}\Xi_B &= \Xi_B(\underline{\mathbf{u}}_{\mathcal{S}\cup\mathcal{R}}, \underline{\mathbf{c}}_{\mathcal{S}\cup\mathcal{R}}, \underline{\mathbf{b}}_{\mathcal{S}\cup\mathcal{R}}, \underline{\mathbf{x}}_{\mathcal{S}\cup\mathcal{R}}) \\ &= \Xi_{F_{nc}}(\underline{\mathbf{u}}_{\mathcal{S}}, \underline{\mathbf{u}}_{\mathcal{R}}) \prod_{a\in\mathcal{S}\cup\mathcal{R}} \underbrace{\Xi_{C_a}(\mathbf{u}_a, \mathbf{c}_a)}_{=\mathbb{1}_{\{\mathbf{c}_a = C_a(\mathbf{u}_a)\}}} \\ &\prod_{a\in\mathcal{S}\cup\mathcal{R}} \underbrace{\Xi_{\pi_a}(\mathbf{c}_a, \mathbf{b}_a)}_{=\mathbb{1}_{\{\mathbf{b}_a=\pi_a(\mathbf{c}_a)\}}} \prod_{a\in\mathcal{S}\cup\mathcal{R}} \underbrace{\Xi_{\phi_a}(\mathbf{b}_a, \mathbf{x}_a)}_{=\mathbb{1}_{\{\mathbf{x}_a=\phi_a(\mathbf{b}_a)\}}},\end{aligned} \quad (8)$$

where $\Xi_{F_{nc}}$, Ξ_{C_a}, Ξ_{π_a}, and Ξ_{ϕ_a} represent the characteristic functions of the behavioral model of the global network encoder, the channel encoder, the channel interleaver, and modulator of $a \in \{s, r\}$, respectively. The characteristic function $\Xi_{F_{nc}}$ can be further factorized as

$$\Xi_{F_{nc}}(\underline{\mathbf{u}}_{\mathcal{S}}, \underline{\mathbf{u}}_{\mathcal{R}}) = \prod_{r\in\mathcal{R}} \Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}}, \mathbf{u}_r), \quad (9)$$

where $\Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}}, \mathbf{u}_r)$ represents the characteristic function of the behavioral model of the local network coding function where its behavior is captured by the equations (7). Finally, the characteristic function Ξ_{F_r} can be factorized as

$$\Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}}, \mathbf{u}_r) = \left(\prod_{s\in\mathcal{S}} \underbrace{\Xi_{\Pi_{s,r}}(\mathbf{u}_s, \tilde{\mathbf{u}}_{s,r})}_{=\mathbb{1}_{\{\tilde{\mathbf{u}}_{s,r}=\Pi_{s,r}(\mathbf{u}_s)\}}}\right) \underbrace{\Xi_{\Pi_r}(\mathbf{u}_r, \tilde{\mathbf{u}}_r)}_{=\mathbb{1}_{\{\tilde{\mathbf{u}}_r=\Pi_r(\mathbf{u}_r)\}}} \mathbb{1}_{\{\tilde{\mathbf{u}}_r = \sum_{s\in\mathcal{S}} \oplus \mathbf{G}_{s,r}\tilde{\mathbf{u}}_{s,r}\}}. \quad (10)$$

The factor graph representing the factorization in (8) is given in Fig. 2. Fig. 3 shows an example of the factor graph for $(2, 2, 1)$-OMAMRC that represents the factorization in (9) and (10).

The maximum a posteriori decoding rule is given by

$$\begin{aligned}\hat{u}_{s,k} &= \arg \max_{u_{s,k}\in\mathbb{F}_2} p(u_{s,k}|\mathbf{Y}_d, B) \\ &= \arg \max_{u_{s,k}\in\mathbb{F}_2} \sum_{\sim\{u_{s,k}\}} p(\underline{\mathbf{u}}_{\mathcal{S}\cup\mathcal{R}}, \underline{\mathbf{c}}_{\mathcal{S}\cup\mathcal{R}}, \underline{\mathbf{b}}_{\mathcal{S}\cup\mathcal{R}}, \underline{\mathbf{x}}_{\mathcal{S}\cup\mathcal{R}}, \mathbf{Y}_d)\Xi_B \\ &\overset{(a)}{=} \arg \max_{u_{s,k}\in\mathbb{F}_2} \sum_{\sim\{u_{s,k}\}} p(\mathbf{Y}_d|\underline{\mathbf{x}}_{\mathcal{S}\cup\mathcal{R}})\Xi_B \\ &\overset{(b)}{=} \arg \max_{u_{s,k}\in\mathbb{F}_2} \sum_{\sim\{u_{s,k}\}} \left(\prod_{a\in\mathcal{S}\cup\mathcal{R}} \prod_{k=1}^{N_a} p(\mathbf{y}_{a,d,k}|\mathbf{x}_{a,k})\right) \Xi_B.\end{aligned}$$

(a) Follows from using the Bayes rule, and the assumption that the frames of the sources have uniform priors. (b) Follows from the fact that the modulated sequences of the sources and the relays are mutually independent and the different links between the nodes are memoryless channels. It is impossible to use brute-force techniques to accomplish the previous maximization, hence we resort to a suboptimal iterative message-passing algorithm (Belief Propagation [16]). Messages between the different nodes of the factor graph will be circulated until convergence.

We consider a parallel message passing scheduler over the factor graph, shown in Fig. 2. At each iteration, the scheduler works as follows:

1) Based on the channel observations, All the modulators check nodes are activated simultaneously. They send their messages, through the interleavers check nodes, to the variable nodes $c_{a,k}$, , where $a \in \{s, r\}$, and $k = 1, \ldots, n_a$. The messages are denoted by $\mu_{\phi_s \to c_{a,k}}(c_{a,k})$.
2) The SISO channel decoders of the sources are activated simultaneously and send their messages to the variable nodes $c_{s,k}$, and $u_{s,\ell}$, where $\ell = 1, \ldots, K$, denoted by $\mu_{C_s \to c_{s,k}}(c_{s,k})$, and $\mu_{C_s \to u_{s,\ell}}(u_{s,\ell})$, respectively.
3) The SISO network decoder F_{nc} is activated and sends its messages to the variable nodes $u_{r,\ell}$.
4) The SISO channel decoders of the relays are activated simultaneously and send their messages to the variable nodes $c_{r,k}$, and $u_{r,\ell}$.
5) The SISO network decoder F_{nc} is reactivated and sends its messages to the variable nodes $u_{s,\ell}$.
6) Based on all the received messages by the nodes $u_{a,\ell}$, where $a \in \{s, r\}$, and $\ell = 1, \ldots, K$, hard decisions are made to obtain the estimates $\hat{\mathbf{u}}_a$. Then, CRC checks are performed to extract the correctly decoded frames. Finally, separate network decoding is performed on the correctly decoded messages. In the case of GFNC, Gauss elimination decoding is used. In the case of BI-XOR or XOR, if a

network coded packet of a relay r and $|\mathcal{S}_r|-1$ packets of the sources in $\mathcal{S}_r$ are correctly decoded then all the packets of $\mathcal{S}_r$ are correctly decoded.

7) If the M frames of the sources are correctly decoded or a maximum number of iterations is reached, the iteration process stops.
8) All the variable nodes $c_{a,k}$, where $a \in \{s,r\}$, and $k = 1,\ldots,n_a$, send their messages to the modulation check nodes.

From (4), we notice that the network coding or decoding could be implemented using binary operations (the decomposition of the local network coding function into three functions is not needed any more). This remark facilitates the soft decoding at the relays and the destination.

E. The SISO network decoder

In this section, we detail the messages generated by the SISO netwrok decoder. The messages generated from F_{nc} to $u_{r,\ell}$, where $r \in \mathcal{R}$, and $\ell = 1,\ldots,K$ are given by

$$\mu_{F_{nc}\to u_{r,\ell}}(u_{r,\ell}) = \sum_{\sim\{u_{r,\ell}\}} \sum_{\mathbf{u}_s : s\in\mathcal{S}} \Xi_{F_{nc}}(\underline{\mathbf{u}}_{\mathcal{S}}, \underline{\mathbf{u}}_{\mathcal{R}}) \prod_{(r',\ell')\neq(r,\ell)} \mu_{u_{r',\ell'}\to F_{nc}}(u_{r',\ell'}) \prod_{s\in\mathcal{S}} \prod_{\ell'=1}^{K} \mu_{u_{s,\ell'}\to F_{nc}}(u_{s,\ell'}). \tag{11}$$

Note that if the channel code C_a is systematic then $\mu_{u_{a,\ell}\to F_{nc}}(u_{a,\ell}) = \mu_{\phi_a\to u_{a,\ell}}(u_{a,\ell})\mu_{C_a\to u_{a,\ell}}(u_{a,\ell})$, and $\mu_{u_{a,\ell}\to F_{nc}}(u_{a,\ell}) = \mu_{C_a\to u_{a,\ell}}(u_{a,\ell})$ if the channel code is not systematic. We further simplify (11) by not using the messages of the variable nodes that belong to network coded packet of r and by only considering the check equation provided by r (separately from the other relays). In this case, the messages generated from F_{nc} to $u_{r,\ell}$, are given by

$$\mu_{F_{nc}\to u_{r,\ell}}(u_{r,\ell}) = \sum_{\mathbf{u}_s : s\in\mathcal{S}} \Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}}, \mathbf{u}_r) \prod_{s\in\mathcal{S}} \prod_{\ell'=1}^{K} \mu_{u_{s,\ell'}\to F_{nc}}(u_{s,\ell'}). \tag{12}$$

Similarly, the messages generated from F_{nc} to $u_{s,\ell}$, where $s \in \mathcal{S}$, and $\ell = 1,\ldots,K$, are given by

$$\mu_{F_{nc}\to u_{s,\ell}}(u_{s,\ell}) = \sum_{\sim\{\mathbf{u}_s\}} \Xi_{F_r}(\underline{\mathbf{u}}_{\mathcal{S}}, \mathbf{u}_r) \prod_{\ell'=1}^{K} \mu_{u_{r,\ell'}\to F_{nr}}(u_{r,\ell'}) \prod_{s'\in\mathcal{S}\setminus\{s\}} \prod_{\ell'=1}^{K} \mu_{u_{s',\ell'}\to F_{nr}}(u_{s',\ell'}). \tag{13}$$

IV. Numerical Results

In this section, we focus on the case where the sources and the relays are separate entities, i.e., $\mathcal{S}\cap\mathcal{R} = \{\emptyset\}$. There are of course an infinity of SNR configurations $\gamma_{r,d}, \gamma_{s,r}, \gamma_{s,d}, \gamma_{r,\tilde{r}}$. By arbitrarily choosing few configurations as examples, we want to evaluate the effectiveness of the proposed coding schemes. We fix the number of receive antennas at the relays $m_r = 1$ and we let the number of receive antennas at the destination $m_d \in \{1,2,4\}$. Each message of the source has a length $k = 256$ information bits. 4-QAM with Gray labeling is used at the sources and the relays, i.e., $q_s = q_r = 2$. The sources use identical punctured turbo codes, with coding rate $w_s = 1/2$, made of two 4-states rate-1/2 RSCs encoders with generator matrix $\mathbf{G}_s(D) = \left[1 \quad \frac{1+D^3}{1+D+D^2+D^3}\right]$. The relays use a punctured convolutional code, with coding rate $w_r = 1$ made of a 16-states rate-1/2 RSC encoder with generator matrix $\mathbf{G}_r(D) = \left[1 \quad \frac{1+D^3+D^4}{1+D+D^2+D^3+D^4}\right]$. The systematic bits are punctured completely. The proposed constituent codes are given only as an example to illustrate the benefits of our approach.

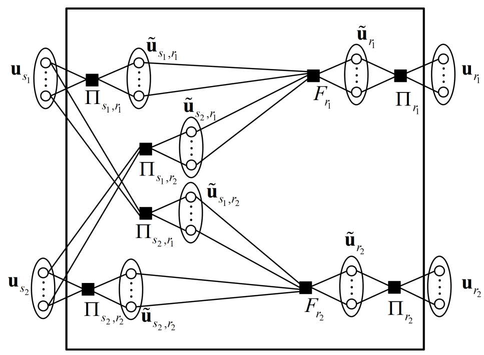

Fig. 3. Factor graph of F_{nc} of $(2,2,1)$-OMAMRC.

In the first simulations, we consider $(2,2,1)$-OMAMRC with imperfect S-R links. For $m_d = 1$, we choose $\gamma_{r,d} = \gamma_{s,r} = \gamma_{s,d} = \gamma_{r,\tilde{r}} = \gamma$. For $m_d = 4$, we take $\gamma_{r,d} = \gamma_{s,d} = \gamma_{r,\tilde{r}} = \gamma$, and $\gamma_{s,r} = 100\gamma$. Fig. 4 shows the results. We observe that

- the GFNC achieves the promised full diversity,
- the BI-XOR has a very close performance to GFNC and starts to deviate at high SNR,
- as expected, the XOR network coding, where the relays directly perform XOR operations on the bits of the correctly decoded frames of the sources without interleaving, does not achieve the full diversity,
- GFNC in JNCC/JNCD framework has a coding gain of 3 dB in the case of $m_d = 1$ and 4 dB in the case of $m_d = 4$ with respect to the GFNC in SNCC/SNCD framework.

similar observations were made for higher modulation, up to 64-QAM, and higher coding rates, up to $5/6$ at the sources. Results are not shown due to space limitation.

In the second simulations, we want to further investigate the performance of BI-XOR. We chose $(2,3,1)$-OMAMRC with $\gamma_{s,r} = \gamma_{s,d} = \gamma_{r,d} = \gamma$, $m_d = 1$, and $m_d = 2$. Fig. 5. shows the IBLER at each relay and the destination when (1) $\gamma_{r,\tilde{r}} = 10\gamma$, and (2) $\gamma_{r,\tilde{r}} = 0$. We observe that

- when BI-XOR is used the diversity is increased with the index of the relays, since relays with high index can listen to relays with lower index, and the destination has the highest diversity, since it can listen to all the relays. Note that if XOR is used at the relays then, the diversity order can not increase with the number of relays.
- the quality of R-R links increases the coding gain at the destination but not the diversity order.

In the third simulations, we benefit from the flexibility and simplicity of the BI-XOR to compare the IBLER of three OMAMRCs with the same sum spectral efficiency MR,

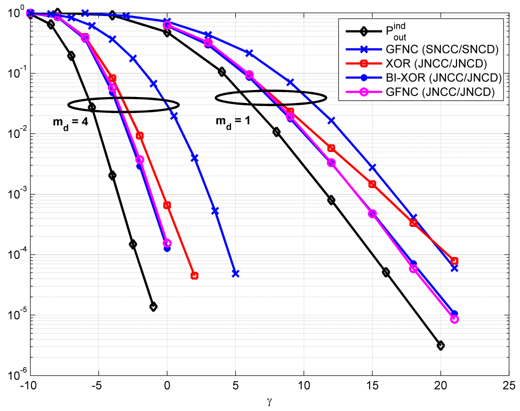

Fig. 4. IBLER vs individual outage probability for (2,2,1)-OMAMRC, $R = 1/3$ (b./c.u), $\alpha = 2/3$.

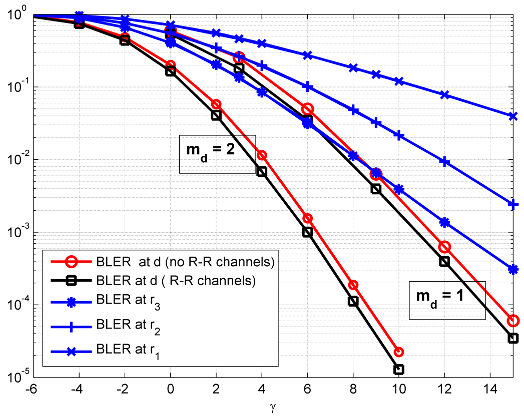

Fig. 5. IBLER at the destination and the relays for $(2, 3, 1)$-OMAMRC, $R = 2/7$ (b./c.u) , $\alpha = 4/7$.

namely $(2, 1, 1)$-OMAMRC, $(4, 2, 1)$-OMAMRC, and $(8, 4, 1)$-OMAMRC. As a benchmark, we calculate the IBLER of GFNC when SNCC/SNCD framework is used. Although the three networks have the same spectral efficiency, it is clear that the last network has the highest possible diversity. We chose $m_d = 1$, and $\gamma_{r,d} = \gamma_{s,r} = \gamma_{s,d} = \gamma_{r,\tilde{r}} = \gamma$. Fig. 6 shows the results. Again we see that the diversity order of BI-XOR increases with the number of the relays. BI-XOR does not achieve the full diversity, except for $(2, 1, 1)$-OMAMRC. Nevertheless BI-XOR has a better performance than the GFNC with SNCC/SNCD at low to moderate SNR. Furthermore, from the result of the first simulation, we could conjecture that in this rage of SNR the GFNC with JNCC/JNCD (which is very complex to implement for $(8, 4, 1)$-MAMRC) will have a very close performance to BI-XOR.

V. Conclusion

In this paper, we have introduced a JNC distributed code design for the orthogonal multiple-access half-duplex relay channel, which is flexible in terms of coding and modulation scheme, as well as, the number of sources and relays. The design is based on the selective relaying approach that depends on the

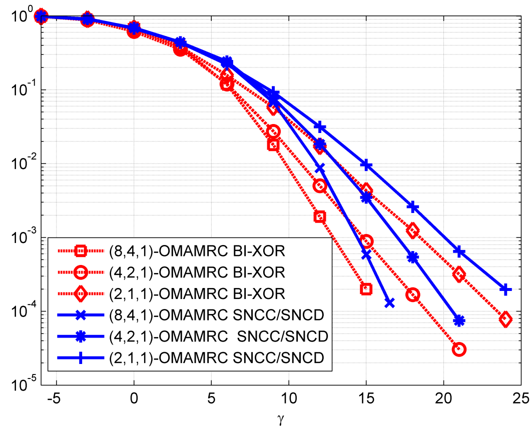

Fig. 6. IBLER of BI-XOR for different OMAMRCs, where $MR = 0.8$ (b./c.u), $\alpha = 0.8$, compared to IBLER of GFNC in SNCC/SNCD framework.

number of correctly decoded messages at each relay. We have described in details the structure of the encoders, when and how JNC coding is performed, and the structure of the JNC decoders. Semi-orthogonal and non-orthogonal MAMRC access schemes represent an interesting future research topics.

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